Smooth splines on unstructured quadrilateral and hexahedral meshes

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ABSTRACT

CAD representations of arbitrary genus geometries with a finite number of tensor-product polynomial patches invariably lead to surface representations with unstructured quadrilateral meshes and, thus, extraordinary vertices. Construction of smooth splines over such unstructured quadrilateral meshes is of considerable interest within the field of isogeometric analysis [1] and a myriad of approaches have been explored [2,5,6] that focus on the design and analysis of such geometries. An even bigger challenge lies in the construction of smooth volumetric representations (required in arterial blood flow simulations [3], for instance) from smooth surface representations.

Recently, a mathematically rigorous approach towards generation of quadrilateral and hexahedral meshes for objects of arbitrary genus was presented in [4]. Building on top of [4], we present an approach towards construction of smooth bi- and tri-variate splines over unstructured quadrilateral and hexahedral meshes, respectively. Our approach employs singular parameterizations at extraordinary vertices inspired by [5,6]. The constructed spline spaces demonstrate superior approximation behavior, and seem to be well behaved even at the singularities. Examples of applications in design and analysis will be presented.

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