Isogeometric Modeling of Red Blood Cells and their Coupling with Plasma through Immersed Methods

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ABSTRACT

We consider the modeling of red blood cells (RBCs) immersed in the plasma. Indeed, when considering small capillaries, for which the size of the vessel is comparable to the size of the RBC, the blood rheology is mostly determined by the dynamics of the RBCs, which outnumber the other cells and platelets present in the plasma. We treat the lipid bilayer biomembrane of the RBC as a surface in the 3D space, whose behavior is characterized by the minimization of the Canham–Helfrich energy. The surrounding plasma is modeled as a Newtonian incompressible fluid, governed by the (Navier–)Stokes equations at low Reynolds numbers.

We discretize the coupled RBC-fluid system of PDEs in space by means of NURBS-based Isogeometric Analysis (IGA) [3] in the framework of the Galerkin method, for which we represent the RBC membrane as a smooth single-patch NURBS surface. In this respect, the use of NURBS with high order of continuity leads to an efficient treatment of the high order differential operators involved in the geometric PDE describing the RBC [1, 2] and permits accurate evaluations of geometrical quantities, such as the curvature and the normal. For the RBC-plasma coupling, we propose a non-boundary-fitted discretization of the fluid domain, still represented by NURBS, with respect to the shape of the RBC. In the fluid problem we account for the RBC by means of a resistive approach or through the weak imposition of interface conditions at the immersed interfaces [4]. We discretize the PDEs in time by means of Backward Differentiation Formulas, with extrapolation of the geometric quantities of the RBC surface and semi-implicit treatment of the Navier-Stokes convective term. We show and discuss numerical results obtained in approximating the coupled problem, highlighting the benefits arising from the IGA discretization.

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