A high-order discontinuous Galerkin solver for turbulent incompressible flow with matrix-free implementation

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ABSTRACT

We present a high-order discontinuous Galerkin solver for the incompressible Navier–Stokes equations [1]. For discretization in time, a dual splitting method based on BDF-3 is used [2] that separately advances the convective term, the incompressibility constraint, and the viscous terms. The explicit treatment of the convective term limits the time step size. The step size selection is done adaptively according to the local velocity field in each cell [3]. The flow field is made solenoidal by a projection step. It uses the solution of a pressure Poisson equation forced by the divergence of the intermediate velocity field after the convective step. The final viscous step is performed by a Helmholtz-like equation in the velocity.

The spatial discretization uses the Lax–Friedrichs flux for the convective term and the symmetric interior penalty method for the second-derivative terms in the pressure Poisson equation and the viscous term. For application to implicit large eddy simulation of turbulent flow which is marginally resolved per definition, two stabilization steps are necessary in the DG context. On the one hand, a consistent div-div penalty term is introduced that suppresses local divergence errors of the magnitude of the discretization error. It is similar to the widely used grad-div stabilization in continuous finite elements. On the other hand, inter-element mass conservation is controlled by central fluxes both in the discretization of the divergence term on the right hand side of the pressure equation and in the discretization of the pressure gradient in the projection step.

The linear systems for the pressure Poisson equation and the viscous step are solved by modern matrix-free solvers, including geometric multigrid techniques with Chebyshev smoothing for the former. All matrix-vector products are implemented with fast sum factorization techniques on quadrilaterals and hexahedra [4]. A particular feature of our solver is that the throughput per degree of freedom is almost independent of the polynomial degree, besides being highly competitive at up to a million degrees of freedom processed per core in one time step. Applications of the solver to implicit LES of turbulent channel flow as well as function enrichment in RANS simulations of a flow past a periodic hills show the capabilities of the solver.

REFERENCES

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