Monotone nonlinear finite-volume method for challenging grids

Martin Schneider*, **Bernd Flemisch*** and **Rainer Helmig***

* Department of Hydromechanics and Modelling of Hydrosystems
University of Stuttgart
Pfaffenwaldring 61, 70569 Stuttgart, Germany
e-mail: Martin.Schneider@iws.uni-stuttgart.de, web page: http://www.hydrosys.uni-stuttgart.de/

ABSTRACT

Porous media applications differ from energy storage in the subsurface to nuclear waste storage up to drug transport through human tissue. Modeling such processes results in highly nonlinear partial differential equations, that often have to be solved on unstructured and non-conforming grids, to fulfill geometric constraints. Due to the additional requirement of mass-conservation, the most commonly used methods for solving flow problems in porous media are Multi-Point Flux Approximation methods, Mixed Finite Element methods, or Mimetic Finite Difference methods. In terms of accuracy, these schemes converge at least with second-order for the considered variable and first order for the fluxes. That means, they are exact for linear and piecewise linear solutions. The major drawback of these schemes is the fact that they are not unconditionally monotone, which means that maximum and minimum principles are not satisfied for arbitrary grids and permeability tensors. Non-monotone schemes may produce unphysical solutions, which in turn influence the efficiency and convergence of the scheme. It is proven that there exist no linear higher order unconditionally monotone control volume schemes.

Relaxation of this linearity requirement allows the construction of monotone schemes. The first idea of a nonlinear scheme that is monotone on triangular grids has been published by Le Potier [1]. Recently, this scheme has been extended to polygonal meshes [2]. Most of the existing literature about nonlinear finite-volume schemes focuses on linear elliptic equations. Only a few publications exist that consider multi-phase flow in porous media [3, 4].

We will present a numerical analysis of accuracy and efficiency of a nonlinear finite-volume scheme for flow in porous media. Furthermore, the influence of nonlinear schemes on the numerical behavior will be shown in detail. We demonstrate that these nonlinear flux approximations may positively influence the convergence behavior of nonlinear and linear solvers, due to the fact that condition numbers of occurring Jacobian matrices are reduced compared to linear methods. Finally, we will exhibit that these schemes can solve benchmark problems as accurate as linear methods.

REFERENCES

[1] Le Potier, C. (2005). *Finite volume scheme for highly anisotropic diffusion operators on unstructured meshes.* Comptes Rendus Mathematique, 921-926.

[2] Lipnikov, K., Svyatskiy, D. and Vassilevski, Y. (2009). *Interpolation-free monotone finite volume method for diffusion equations on polygonal meshes.* Journal of Computational Physics, 228(3), 703-716.

[3] Nikitin, K., Terekhov, K. and Vassilevski, Y. (2014). *A monotone nonlinear finite volume method for diffusion equations and multiphase flows.* Computational Geosciences, 18(3-4), 311-324.

[4] Schneider, M., Flemisch, B. and Helmig, R. (accepted 2016) *Monotone nonlinear finite-volume method for nonisothermal two-phase two-component flow in porous media.* International Journal for Numerical Methods in Fluids.