Orthogonal Basis Bubble Function Finite Element Method for Shallow Water Equations and Boussinesq Equations

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ABSTRACT

In this paper, a bubble function element stabilization method [1] for nonlinear shallow water long wave equations and Boussinesq equations is proposed. Special bubble function is extended as an orthogonal basis bubble function element stabilization method [2] for P1b element.

An important point to be noted is that the consistent mass matrix is a diagonal matrix on account of the orthogonal intersection of the basis functions of orthogonal basis bubble function element. Explicit and implicit finite element methods with orthogonal basis bubble function element are proposed in nonlinear shallow water long wave equations. In explicit scheme, the results of the diagonal mass matrix using orthogonal basis bubble function element are in better agreement with exact solutions of hydraulic jump problem [3] and wind-driven current problem [4] than the results of the lumped mass matrix using linear bubble function element. In implicit scheme, the orthogonal basis bubble function element stabilization method obtains better numerical stability and computational accuracy than classical bubble function element.

Implicit finite element method with orthogonal basis bubble function element is proposed in Boussinesq equations. The results using the orthogonal basis bubble function element stabilization method are in better agreement with experiment data of run up problem of the solitary wave against a vertical wall [5,6] than the results using classical bubble function element.

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