A simple robust and accurate a posteriori sub-cell finite volume limiter for the Discontinuous Galerkin method

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ABSTRACT

In this talk we present a novel simple, robust and accurate nonlinear *a posteriori* stabilization technique of the Discontinuous Galerkin (DG) finite element method for the solution of nonlinear hyperbolic PDE systems on general *unstructured* triangular and tetrahedral meshes in two and three space dimensions, as well as on uniform and space-time adaptive Cartesian grids. This novel approach, which has been recently proposed by Dumbser et al. in a series of papers [1-4], is able to resolve discontinuities at a sub-grid scale and works also for very high polynomial approximation degrees in two and three space dimensions. It can be summarized as follows: At the beginning of each time step, an approximation of the local minimum and maximum of the discrete solution is computed for each cell, taking into account also the vertex neighbors of an element. Then, an *unlimited* discontinuous Galerkin scheme of approximation degree N is run for one time step to produce a so-called *candidate solution*. Subsequently, an *a posteriori* detection step checks the unlimited candidate solution at time t^{n+1} for positivity, absence of floating point errors and whether the discrete solution has remained essentially within the bounds given by the local minimum and maximum computed in the first step. Elements that do not satisfy all the previously mentioned detection criteria are flagged as troubled cells. For these troubled cells, the candidate solution is *discarded* as inappropriate and consequently needs to be *recomputed*.

Within these troubled cells the old discrete solution at the previous time t^n is scattered onto $N_s=2N+1$ smaller sub-cells per element edge, in order to obtain a set of sub-cell averages at time t^n. Then, a more robust second order TVD finite volume scheme or a more accurate ADER-WENO finite volume scheme is applied to update the sub-cell averages within the troubled DG cells from time t^n to time t^{n+1}. The new sub-grid data are finally gathered back into a valid cell-centered DG polynomial of degree N by using a classical conservative and higher order accurate finite volume reconstruction technique. The new limiter can also be interpreted as an element local checkpointing and restarting of the solver, where the restart uses a more robust scheme on a finer grid. A completely new and very peculiar feature of our new limiter for DG schemes is its ability to detect and even *cure* floating point errors (NaN) during the simulation. At shock waves, the resulting method is as robust as standard second order TVD schemes, and it is at the same time as accurate as high order unlimited DG schemes in smooth regions. It is furthermore *positivity preserving* for suitable subgrid schemes.

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