Adaptive Discontinuous Galerkin Methods for flow in porous media

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ABSTRACT

We present an adaptive Discontinuous Galerkin discretization for the solution of porous media flow problems. We consider a strongly heterogeneous porous medium and discontinuous capillary pressure functions. We write the system in terms of a phase-pressure/phase-saturation formulation. First and second order Adam-Moulton time discretization are combined with various interior penalty DG discretizations in space such as the Symmetric Interior Penalty Galerkin (SIPG), the Nonsymmetric Interior Penalty Galerkin (NIPG) and the Incomplete Interior Penalty Galerkin (IIPG) [1]. This implicit space time discretization leads to a fully coupled nonlinear system requiring to build a Jacobian matrix at each time step for the Newton-Raphson method. The adaptive approach implemented allows for refinement/coarsening in both the element size and the polynomial degree. To our knowledge, this is the first time the concept of local *hp*-adaptivity is incorporated in the study of *2d* and *3d* incompressible, immiscible, two-phase flow problems. The implementation is based on the Open-Source PDE software framework DUNE [2].

REFERENCES

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