Discontinuous Galerkin-type High-Order Numerical Scheme for Vortex Sheet Intensity Computation in Two-Dimensional Vortex Method

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ABSTRACT

Vortex methods are lagrangian numerical method, which are suitable for solving number of engineering problems connected with incompressible flow simulation. The primary variable in vortex method is vorticity and when its distribution is known, all other flow parameters (velocity and pressure) can be reconstructed by using Biot — Savart law and Cauchy — Lagrange integral analogue. In the present research two-dimensional problems are considered. In order to simulate how the airfoil influences the flow, it should be replaced by vortex sheet of unknown intensity which is placed on the camber line. Its intensity can be found from solution of the integral equation.

Normally in vortex methods low-order numerical schemes are used for solving of the integral equation. In the most accurate schemes which are known nowadays, the solution is considered to be only piecewise constant along the panels — segments, which approximate camber line of the airfoil. These segments normally are rectilinear. In order to raise the accuracy, discontinuous-Galerkin (DG)-type approach can be implemented. The main difficulty is that the matrix of the corresponding linear system is not sparse (as in case of solving differential equation), and in order to calculate its coefficients, not very simple integrals should be computed. Their numerical computation is also non-trivial problem, because sometimes they are improper or singular.

Firstly, the solution can be considered to be piecewise-linear along the panels, so 2 shape functions on every panel are introduced. For this case the exact analytical formulae are derived for all matrix coefficients. However, this approach provides not more than 2nd order of accuracy in C (for smooth airfoils) and L_1 norms. In order to take into account the curvilinearity of the camber line of the airfoil, the more accurate approach is developed: for camber line Hermitian polynomial (cubic) interpolation is used and three shape functions are used for solution approximation: constant, preorthogonalized linear and quadratic ones. For such approach approximate analytical formulae are obtained, where 2 small parameters are introduced, which connected with the panel's shape. Only terms which are linear with respect to these parameters are taken into account, however, such accuracy is quite enough for practical purposes.

The important feature of DG-type approach in vortex methods is that it allows to approximate numerically solution for vortex sheet intensity which is discontinuous, that is especially important for simulation of the flows around airfoils with sharp edge or angle point by using vortex methods. High-order shape functions usage both for solution and for the camber line shape approximation makes it possible to take into account the curvilinearity of the airfoil, that is important, for example, in simulation of the flows with free surface.

REFERENCES

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