A priori error-based mesh adaptation for CFD

Éléonore Gauci[+], Anca Belme[†], Adrien Loseille[‡], Frédéric Alauzet[‡] and Alain Dervieux[*]

[+*] Ecuador, INRIA, 2002 route des lucioles,
F-06902 Sophia-Antipolis
e-mails: eleonore.Gauci@inria.fr, alain.dervieux@inria.fr

[†] Université Pierre et Marie Curie
Institut Jean le Rond d'Alembert, Tours 55-65, Bureau 424, Case 162
4 Place Jussieu, F-75452 Paris
e-mail: belme@dalembert.upmc.fr

[‡] Gamma3, INRIA, 1 rue Estienne d'Orves,
F-91126 Palaiseau
e-mails: adrien.loseille@inria.fr, frederic.alauzet@inria.fr

ABSTRACT

The proposed communication focuses on adjoint-based and metric-based anisotropic mesh adaptation methods. A family of mesh adaptation methods starts with the mathematical formulation of a minimum problem for a functional j depending on the approximation error $u - u_h$. The central issu is then to express this functional in terms of the metric (representing the mesh), in such a way that an optimal metric can be optained from the optimality condition $j' = 0$ for the functional j. A difficult point is to take into account the anisotropy. Several ideas are proposed to attack this point. Let us mention an *a priori* analysis relying on element-mapping proposed in [2]. In [1], a metric optimization is performed from local perturbation of the mesh and of the solution.

In the proposed work, we try to rely on interpolation errors. Indeed, the approximation error of many EDP discretization can be at least formally expressed in terms of interpolation errors of terms of the residual. This is true in particular for continuous and discontinuous FEM. In turn, these interpolation errors are easy to express in terms of the local properties of the mesh, and in particular the mesh metric represented by a positive symmetric matrix field.

Although not a systematic approach, the proposed analysis can be applied to a large number of physical models discretization. The communication will present a sample of such analyses for CFD models, from Poisson equation to compressible Navier-Stokes. Further, we report on the implementation of these analyses. Three families of criteria are compared, the Hessian-based criterion, the goal-oriented criteria, and the norm-oriented criteria. We show that the new alayses carry important improvements as compared with a more classical method, like the direct Hessian-based ones.

REFERENCES

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