Level-Set treatment for an Extended DG Method

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ABSTRACT

We are going to present a high-order discretization for two-phase flow problems, based on an eXtended Discontinuous Galerkin (XDG) discretization. It combines a classical DG method with an interface treatment similar to the extended Finite Element methods (XFEM). This achieves the 'h^p-convergence' property of the DG method even for low-regularity, discontinuous solutions.

The position of the fluid interface is determined by the zero-isocontour of a level-set φ. This requires additional care:

First, the level-set is defined in the whole domain, but the interface velocity is physically defined at the interface only. Since the evolution of the level-set field has to be computed in the entire domain, a continuation of this velocity into the domain is required. For example in solidification, the velocity is defined by the jump in heat flux across the interface. In multiphase flows the velocity field may have a kink due to a jump in the viscosity, or even a jump due to mass flux across the interface. Thus, the velocity value at the interface has to be extended from the interface into the domain.

Second, variations in the level-set gradient must be well behaved: an overly large gradient causes errors during the advection of the level-set, an overly flat gradient causes difficulties in detecting the interface. Thus, we use reinitialization to ensure the signed-distance property, i.e. $\|\nabla\varphi\| = 1$.

For reinitialization and extension, we will compare geometric and PDE-based approaches, see [1].

Third, the surface tension depends on the curvature of the interface. This curvature is computed from first and second derivatives of the level-set function, which are in general not continuous. We present a stabilized curvature evaluation process, see [2]. The algorithm is based on a patch-recovery process, to regain approximate continuity of higher derivatives of φ.

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