Stability analysis of a free-surface problem with moving contact line

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ABSTRACT

The simulation of free-boundary problems is of major relevance in many fluid-dynamics applications, both at the big scale, like in the study of ocean waves and the design of watercraft, and at the microscopic scale, e.g. in the microfluidics of capillary networks or labs-on-a-chip. In these settings, the interaction between different phases requires to correctly track the evolution of the interfaces and of their intersections, the so-called contact lines.

In our work, we analyze a free-surface problem for a Newtonian fluid inside a capillary tube, described by time-dependent, incompressible Navier-Stokes equations. The fluid is in contact with a gas and a solid wall, thus the boundary conditions include the effects of surface tension and wall friction. In particular, the generalized Navier boundary condition [1, 2] is employed to impose an equilibrium contact angle, by setting a co-dimension-2 force field concentrated on the contact line. The equations governing the phenomenon are motivated and derived from physical variational principles, mainly hinging upon the Principle of Minimum Reduced Dissipation [3]. The system is then discretized by means of the Arbitrary Lagrangian-Eulerian approach. The numerical properties of the resulting scheme are investigated, drawing a parallel with the physical properties holding at the continuous level. Some instability issues are addressed in detail, in the case of an explicit treatment of the geometry, and a novel additional stabilization term is introduced in the discrete formulation in order to dampen the instabilities arising at the free surface. Numerical tests assess the suitability of the numerical scheme, the influence of the parameters on the solution, and the effectiveness of the new stabilization.

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