Maximum-principle-satisfying space-time conservation element and solution element scheme for solving compressible multifluids

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ABSTRACT

An important property of the entropy solution of scalar conservation laws is satisfying a strict maximum (or bound-preserving) principle [1], that is, if the initial value of the conservative quantity is in the range of [m, M], the solution will remain in this range at any time and position. Numerical schemes with this property are very useful for some applications (e.g., flows consisting of several fluid components). The space-time conservation element and solution element (CE/SE) scheme [2, 3] can capture shocks with high resolution, but it does not satisfy maximum-principle. To date, no maximum-principle-satisfying CE/SE scheme has been proposed. In this work, a maximum-principle-satisfying CE/SE scheme is constructed to solve a reduced five-equation model coupled with the stiffened equation of state for compressible multifluids. We first derive a sufficient condition for CE/SE schemes to satisfy maximum-principle when solving a general conservation law. And then we introduce a slope limiter to ensure the sufficient condition which is applicative for both central and upwind CE/SE schemes. Finally, we implement the upwind maximum-principle-satisfying CE/SE scheme to solve the volume-fraction-based five-equation model for compressible multifluids. A number of numerical examples are carried out to carefully examine the accuracy, efficiency, conservativeness and maximum-principle-satisfying property of the proposed approach.

REFERENCES

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