Ensured Energy Decay in Stabilized Methods using Dynamic Orthogonal Subgrid-Scales and Isogeometric Analysis

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ABSTRACT

Energy inconsistency in the numerical formulation of partial differential equations could lead to incorrect results [1]. Examples of incorrect behaviour caused by energy inconsistencies can occur when solving two-fluid flow. Augmenting an algorithm for the evolution of the two-fluid interface with a standard incompressible Navier-Stokes solver in a monolithic fashion has several advantages.

This work is devoted to the construction of a correct-energy-decaying stabilized finite element method. Stabilized methods and multiscale formulations form an auspicious, versatile and fundamental class of methodologies for finite element computations of complex flow problems and turbulence. The classical Galerkin variational formulation depicts correct energy behavior although it has limitations concerning accuracy and stability. The popular stabilized methods, i.e. the Streamline upwind Petrov-Galerkin method (SUPG), the Galerkin least-squares method (GLS), and the variational multiscale method (VMS), overcome these issues, however do not show correct energy behavior. Our contribution corrects this deficiency by employing the idea of dynamic orthogonal subgrid-scales within the framework of stabilized methods. The multiscale stabilization method based on orthogonal subgrid-scales, proposed by Codina [2], is a key ingredient of the approach. We combine this methodology with residual-based variational multiscale turbulence modeling, a concept which emanates from VMS. The dynamical behavior of the orthogonal subgrid-scales allows for global momentum conservation and suits the turbulence computations [3]. The above is implemented using div-conforming isogeometric analysis (IGA) spaces. The IGA concept, a new computational technology proposed by Hughes and collaborators in [4], is superior to finite elements in terms of accuracy.

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