

# An extended Discontinuous Galerkin method for transient two-phase problems: Application in gravity/surface tension driven flows

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## ABSTRACT

We are going to present a high-order numerical method for transient two-phase flow problems, based on a Discontinuous Galerkin (DG) discretization. Two-phase problems introduce due to the discontinuity of the fluid properties at the interface, kinks in the velocity field and jumps in the pressure field. In order to preserve the  $h^{p+1}$  spatial convergence for such low-regularity solutions, the approximation space on cut cells is adapted to the position of the interface. The interface is described by the zero-isocontour of a level-set function [1].

Considering time-dependent problems, the coupling of the interface evolution and the flow solver is crucial and thus achieving high-order convergence in time is challenging. Reasons for that are changing approximations spaces with vanishing and new generated cut cells due to the movement of the interface over cell boundaries. Recently a moving mesh approach was adapted for the time integration of extended DG methods. A comparison to classical splitting techniques, such as Lie and Strang splitting, is investigated and convergence studies in time will be shown.

The investigations will focus on transient flows driven by gravity and surface tension forces, such as the Rayleigh-Taylor instability and the rising bubble. In such flows special care must be taken of the capillary timestep restriction and the computation of the curvature, which introduces surface tension forces. Both issues are liable for numerical instabilities. The level-set function and thus the curvature are in general not continuous after the evolution. Therefore we will present a stabilized curvature evaluation process [2]. This algorithm adapts a patch recovery filtering technique, which is based on a L2-projection, in order to regain continuity for higher derivatives of the level-set function.

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