A Conservative, Optimization-Based Semi-Lagrangian Spectral Element Method for Passive Tracer Transport

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ABSTRACT

We present a new optimization-based, conservative, and quasi-monotone method for passive tracer transport. The scheme combines a high-order spectral element semi-Lagrangian (SL-SEM) scheme with a novel, optimization-based strategy [1] for the enforcement of the relevant physical properties, such as mass conservation and physically motivated solution bounds. Specifically, the raw high-order solution of the SL-SEM, obtained by backtracking the integration points and then using the given spectral element basis on the departure elements, defines an optimization target. On the other hand, mass conservation and physically motivated local solution bounds provide the optimization constraints. The actual solution is then determined by solving a singly linearly constrained quadratic program with simple bounds, which can be done very efficiently using the recently developed fast, optimization-based remap algorithm [2].

The scheme can handle efficiently a large number of passive tracers because the semi-Lagrangian time stepping only needs to evolve the grid points where the primitive variables are stored and allows for larger time steps than a conventional explicit spectral element method. Numerical examples show that the use of optimization to enforce physical properties does not affect significantly the spectral accuracy for smooth solutions. Performance studies reveal the benefits of high-order approximations, including for discontinuous solutions.

We also briefly comment on the extension of these ideas to more general transport problems involving non-solenoidal velocity fields.

REFERENCES

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