Discontinuous Petrov Galerkin (DPG) Method with Optimal Test Functions with Application to Coupled Wave Propagation Problems

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ABSTRACT

We will present our most recent results on the DPG method for wave propagation problems. The essence of the method lies in a combination of two ideas: use of approximate optimal test functions computed on a fly, and use of broken (discontinuous) test spaces that makes the determination of optimal test functions feasible. The methodology is definitely significantly more expensive than standard finite elements, we roughly double the number of interface unknowns, and the element computations are much more expensive.

On the other side, we gain a lot. Being a Ritz method, DPG does not suffer from any pre-asymptotic instabilities. The method comes with an-posteriori error estimate built it, and provides a very natural setting for adaptivity. For wave propagation problems, standard Galerkin method is only asymptotically stable. To guarantee the stability, we need to start with a mesh that not only satisfies the Nyquist criterion (controls the best approximation error) but also controls the pollution (phase error). For large wave numbers, this translates into using enormous size meshes. In contrary, the DPG method may start with a very coarse mesh that captures geometry only, with the ultimate mesh built through the adaptive process.

The presentation will cover recent theoretical results obtained in [1], as well as a number of numerical wave propagation examples focusing on problems where the solution is very localized. To this class belongs a large class of coupled acoustics/elastodynamics problems relevant to modelling of sonic tools in borehole simulations [2].

REFERENCES

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