A Variational, FIC-Based Variational Formulation For Fluid-Structure Particle Finite Element Methods

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ABSTRACT

Finite Calculus (FIC) has been used as stabilization tool for computational methods in science and engineering applications since 1998 [1]. It has been particularly successful for modeling and simulation of advective flow processes. Because such transport models lie outside the framework of classical variational calculus, the spatial discretization task to obtain finite element equations has relied on weighted residual methods.

Recent success in Particle Finite Element Methods (PFEM) for fluid and fluid-structure interaction problems [2] has opened up the possibility of devising a unified FIC variational formulation that encompasses solid and fluids, whether compressible or incompressible. This unification becomes feasible because the Lagrangian kinematic description of PFEM eliminates advective terms. A common formulation offers advantages in software reuse, since discarding the stabilization terms for compressible solids reduces the FIC functional to conventional forms. Thus existing FEM libraries for linear and nonlinear analysis could be utilized without change for modeling the compressible part of fluid-structure interaction problems.

We have developed a variational framework that encompasses both fluids and solids, and accommodates incompressible and near-incompressible material behavior. The main result is the development of a modified mixed functional that has displacements and pressure as primary variables and incorporates FIC steplengths suitable for stabilizing incompressible behavior [3]. The Euler-Lagrange equations of the functional agree with FIC modified residual equations written with respected to centered points chosen over balance-control domains. Displacement-pressure finite elements developed under this framework are tested in one dimensional static and dynamic benchmark problems. Static benchmark solutions are shown to be nodally exact in both displacements *and* pressures if appropriate continuity rules at material interfaces are obeyed.

The spectral dynamic behavior of the formulation has been studied for regular 1D, finite and infinite lattices with and without material interfaces. While the value of the FIC stabilization steplength is not relevant to static problems — as long as that value is nonzero, and interface pressure discontinuities are correctly accounted for — in dynamic analysis both magnitude and sign of that parameter turn out to be important. Dynamic stability regions have been identified and displayed on the ν (Poisson's ratio) versus τ (dimensionless FIC steplength) plane. They have been verified in benchmark dynamic problems treated by direct time integration.

The paper concludes with the analysis of direct time integration methods to solve the dynamic problem. Both monolithic and partitioned (staggered) treatments are considered. In the later, displacements (or velocities) are advanced separately and a corrective iteration is optionally applied. The conditions for A-stability are analyzed for both treatments. It is found that for achieving A-stability of the staggered approach, an augmentation of the pressure equations by a diagonal matrix is required. The analysis is corroborated with numerical experiments.

References

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