A Reduced Basis Method for heterogeneous domain decomposition problems

I. Martini*, G. Rozza† and B. Haasdonk*

* Institute of Applied Analysis and Numerical Simulation (IANS)
University of Stuttgart
Pfaffenwaldring 57, 70569 Stuttgart, Germany
Email: {immanuel.martini,haasdonk}@mathematik.uni-stuttgart.de

† SISSA MathLab
International School for Advanced Studies
Via Bonomea 265, 34136 Trieste, Italy
Email: gianluigi.rozza@sissa.it

ABSTRACT

The Reduced Basis method allows efficient model reduction of parametrized partial differential equations. It is a snapshot-based method that builds upon a high-dimensional “detailed” approximation method, such as the Finite Element or the Finite Volume method. By the widely used Greedy-algorithm a set of sample solutions is generated, that spans the Reduced Basis approximation space. After that, a Galerkin projection yields the reduced solution. The efficiency of the method highly relies on a decomposition of the computations into an offline- and online-phase and on a-posteriori error estimates, that allow to estimate the error induced by the reduction.

A critical issue in Reduced Basis methods for domain decomposition problems is to find functions that approximate the traces of the detailed solutions on the interface. There have been several efforts including an eigenvalue decomposition [1], “empirical” basis functions [2] and “fourier” interface functions [3]. In our work we consider a framework, that treats the approximation on the interface as a black box. In this way, we can treat heterogeneous domain decomposition problems as opposed to previous works. We also present a-posteriori error analysis for problems that include coercive as well as non-coercive subproblems.

Numerical results are demonstrated with a coupled Stokes-Darcy model as a scenario for a groundwater application. We demonstrate the flexibility of the method with respect to the number of interface basis functions and show that our approach yields an efficient and accurate approximation if few global “detailed” solutions are computed in the offline-phase. We also observe, that the a-posteriori error estimation can be problematic when dealing with heterogeneous problems.

REFERENCES

[1] D.B.P Huynh, D.J. Knezevic and A.T. Patera, “A static condensation reduced basis element method: Approximation and a posteriori error estimation”, *M2AN*, Vol. **47(1)**, pp. 213—251, (2013).

[2] J.L. Eftang and A.T. Patera, “Port reduction in parametrized component static condensation: approximation and a posteriori error estimation”, *IJNME*, Vol. **96(5)**, pp. 269—302, (2013).

[3] L. Iapichino, “Reduced basis method for the solution of parametrized PDEs in repetitive and complex networks with application to CFD”, Ph.D. Thesis, *EPFL, Lausanne,* (2012).

[4] I. Maier and B. Haasdonk, “A Dirichlet-Neumann reduced basis method for homogeneous domain decomposition problems”, *APNUM*, Vol. **78**, pp. 31—48, (2014).

[5] I. Martini, G. Rozza and B. Haasdonk, “Reduced basis approximation and a-posteriori error estimation for the coupled Stokes-Darcy system”, Preprint, *SRC SimTech*, *University of Stuttgart*, (2014).