Simulations of Bubble Collapse in a Non-Linear Viscoelastic Medium

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ABSTRACT

Understanding the mechanics of shock waves in viscoelastic media is important for various naval and medical applications, particularly in the context of cavitation-induced damage. Two examples are erosion to drag-reducing elastomeric coatings along propellers and therapeutic ultrasound such as histotripsy. Histotripsy uses high-amplitude (100 MPa peak positive pressure, -25 MPa peak negative pressure) and high-frequency (MHz) ultrasound waves to destroy tissue. The local/transient pressure changes may lead to the formation of cavitation bubbles that grow and violently collapse, thus producing shock waves that propagate in the surroundings. Although not fully understood, the damage mechanism combines the effect of the incoming pulses and cavitation (bubble oscillation and collapse) produced by the high tension. In such problems, the constitutive models describing the material are non-trivial and include effects such as (nonlinear) elasticity, history and viscosity. Thus, the influence of the shock on the material and the response of the material to the shock are poorly understood. Understanding these mechanisms will provide invaluable insights necessary to fully develop tissue cancer therapies.

A novel numerical approach is proposed for simulating shock and acoustic wave propagation in a non-linear viscoelastic medium [1]. In this approach, an objective time derivative is taken of the constitutive relation that models the material of interest using strain rates and strains. This operation changes the Langragian strains to strain rates and, as a consequence, an additional set of equations for the shear rate tensor that are in an Eulerian framework suitable for shock-capturing methods. In this work, tissue is modelled using the linear Standard Linear Solid model and the non-linear, hyperelastic Neo-Hookean material model. A three-dimensional Eulerian finite-difference method based on a high-order accurate weighted essentially non-oscillatory (WENO) scheme for shock capturing is implemented [2]. For time integration, explicit fourth-order Runge-Kutta is used. The HLL Riemann solver is used for the primitive variables [3]. For the right-hand source derivative terms, WENO reconstruction is performed.

To study these problems in a general sense, a canonical problem is considered, which involves a wave-induced collapse of a gaseous bubble in a viscoelastic medium next to a second viscoelastic or elastic medium of a certain thickness. Results for relevant waveforms that induce the bubble collapse and various "hard" and "soft" tissue-like and elastomeric physical properties will be presented. The stresses, strains and temperatures produced during this process will be presented for different initial stand-off distances, thicknesses and shear moduli of the second medium.

REFERENCES

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