A Path-Conservative Osher-Type Scheme for Simulating Axially Symmetric Compressible Flows in Flexible Tubes

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ABSTRACT

Flexible tubes are widely used in hydraulic systems as connections of components. To design and analyse efficient hydraulic systems, one needs to simulate the flexible tubes as part of such systems. In this paper we want to model the fluid-structure-interaction (FSI) problem given by the axially-symmetric flow of a compressible barotropic fluid that flows through flexible tubes made of vulcanized rubber. The material of the tube can be modelled by using viscoelastic models, which take into account the relaxation of the material. We use a three-parameter Maxwell model to represent the viscoelastic behaviour of the material.

For the fluid flow we assume axial symmetry and a constant (hydrostatic) pressure along the tube radius. As a consequence we consider the one dimensional cross-sectionally averaged compressible Euler equations, which contain some non-conservative terms. In these equations we include the cross-sectional area of the tube as additional unknown like in [5]. The cross-sectional area is linked to the fluid pressure by the three-parameter Maxwell model. The resulting coupled system describes the behaviour of both the fluid and the structure in one set of non-conservative hyperbolic partial differential equations (PDE). This way we realize the fluid-structure-interaction. To solve this non-conservative PDE we employ the family of path-conservative schemes developed by Parés [4] and co-workers. In this particular case here, we use the path-conservative Osher-type Riemann solver as described in [1] and [2] for a hyperbolic system with a conservative and a non-conservative part.

The algorithm is validated by solving several Riemann problems, for which a quasi-exact solution is available. In addition, a comparison to experimental results, presented by Leonhardt [3], is made via a frequency analysis.

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