Observable Euler Equations for Inviscid Regularized Two Phase Flow Simulation

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ABSTRACT

We present an overview of an inviscid regularization technique for conservation laws. The technique is based on the realization that observation of any field quantity is only achievable at a finite scale, here referred to as observability scale α, and not at the mathematical differential limit required in derivation of the governing differential equations; see Mohseni [1]. The main manifestation of this observability scale is in the derivation of the divergence theorem of Gauss. If an infinitely resolved vector field is averaged for scales below α before one take the steps in proving the divergence theorem, one might obtain the observable divergence equation. Systematic application of the Gauss theorem in application of the conservation laws in fluid mechanics results in the derivation of the observable divergence equation, see [1, 2].

This technique has been applied to several classical shock and turbulence problems including Burgers equation, the homentropic Euler equations, and the Euler equations and the results match very well with previous publications. We present some preliminary results from a pseudo-spectral code developed to solve the observable Burgers equation, the observable homentropic Euler equations, and the observable Euler equations. We present Burgers equation with a Gaussian initial condition that quickly develops into a shock. For the homentropic Euler equations, we present a shock tube example that exhibits a shock and an expansion wave. For the Euler equation, we present the Shu-Osher problem: a shock passing through a perturbed density field.

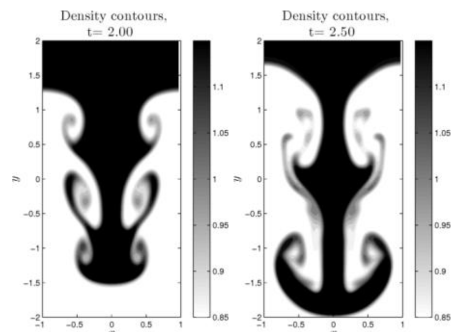

Fig. 1: *Rayleigh-Taylor Instability problem. Simulations using observable Euler equations with no physical viscous*

We are currently investigating the use of the observable Euler equations for the simulation of multiphase flows. In this case, the interface between two fluids is simply treated as a discontinuous change in fluid properties. For example, water has a much higher density and viscosity than air. We present a simulation of 2d Rayleigh-Taylor instability (see Fig. 1) and a 2d two phase flow with an air bubble rising in a quiescent liquid.

Because the observable method presents an inviscid regularization technique, we need to choose a numerical method that does not introduce numerical dissipation and act to regularize the solution. For this reason, we choose to use pseudo-spectral method for all simulations. We may also use finite difference method on a staggered grid with continuum surface force method introduced by J. U. Brackbill [3] to calculate the surface tension force in two phase flow simulations.

REFERENCES

[1] Mohseni, K. “Observable Divergence Theorem: Evolution Equations for Inviscid Regularization of Shocks and Turbulence”, 2010arXiv1010.2612M. http://arxiv.org/pdf/1010.2612v1.pdf

[2] Norgard, G. And Mohseni, K., “A inviscid regularization of the one-dimensional Euler and homentropic Euler equations,” *SIAM Journal, Multiscale Modeling and Simulation.*, Vol. 8, No. 4, 2010, 1212-1242.

[3] J. U. Brackbill, D. Kothe, C. Zemach, “A continuum method for modeling suface tensión”, *J. Comput. Phys*. 100 (1992) 335-353.