

# Finite Element Analysis of Flexo-thermo-piezo-magneto-electricity using Cosserat Micropolar Mechanics

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## ABSTRACT

The use of multi-coupled materials in modern applications is constantly increasing due to their inherent multi-functionality. However, most of these applications use micro-devices and hence, the micro-structure of these materials must be taken into consideration. This poses a challenging problem to the computational mechanics community. The present work extends a 3D Finite Element (FE) formulation [1] to study a fully coupled thermo-electro-magneto-mechanical problem, assuming small strains and adding the micro-structure of the medium. For this purpose, a Cosserat micropolar model is considered: the macro-continuum is simulated by macro-displacements and the micro-continuum by micro-rotations.

Mechanical balance equations to be solved include linear and angular momenta where the latter is included since the stress tensor is non-symmetric due to the inclusion of the micro-continuum. Electrodynamics balance equations include a particular case of the Maxwell laws in order to obtain two equilibrium and two compatibility equations. From the Thermodynamics standpoint, the entropy balance is used in order to evaluate the temperature, where small increments are assumed with respect to a reference state. This consideration leads to a conservative formulation where a Legendre transform can be employed with the purpose of obtaining the constitutive equations. Finally, a set of six fully coupled equations is obtained.

From a numerical point of view, standard first-order isoparametric eight noded elements with nine degrees of freedom per node (three displacements, three rotations, electric potential, magnetic potential and temperature) are used. Time integration is solved by using the Newmark- $\beta$  algorithm and nonlinearities, which arise due to the presence of the Maxwell stress tensor, via the Newton-Raphson method.

Finally, the FE formulation is validated against analytical solutions and convergence patch tests. Since material properties to model the micro-structure are not yet available in the literature, these properties are assumed to be a percentage of the macro-properties. This assumption permits to study the influence of the micro-structure on the output, concluding that the micro-structure plays an important role and must be considered in the study of micro-devices.

## REFERENCES

- [1] J.L. Pérez-Aparicio, R. Palma and R.L. Taylor, Multiphysic and thermodynamic formulations for equilibrium and non-equilibrium interactions: nonlinear finite elements applied to multi-coupled active materials, Archives of Computational Methods in Engineering, to appear 2015.