On SAND Problem Formulations of Structural Topology Optimization With Singular Stiffness Matrices

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ABSTRACT

We consider a class of classical optimization problems arising in the field of topology optimization of mechanical structures and material optimization. We use the well-known approach of optimal material distribution within a 2D or 3D domain, i.e., the local design variables are interpreted as material densities. This approach avoids parametrization of the boundary of the structures, thus working with an extremely rich set of feasible designs. We consider the resulting optimization problem in discrete (resp. discretized) form, i.e., for discrete mechanical structures (as, e.g., trusses) or for continua after finite element discretization. One of the major challenges of the approach is the treatment of the coupling of optimization variables (i.e., design variables) and state variables (e.g., displacements) in the situation where some or many structural/finite elements are assigned to zero density (e.g., for holes in the structure). Of course, this is an inherent difficulty of the approach. For the studied problem class we consider the usual coupling of design and state variables in form of (linear) elastic equilibrium. In terms of mathematics, the above difficulty can be described as the treatment of singular stiffness matrices during the optimization process.

Most numerical approaches work with positive lower bounds on the design variables in a so-called NAND (nested analysis and design) approach. In these approaches zero densities are prohibited, and the practitioner must handle ill-conditioning. In addition, a more serious theoretical difficulty comes in play. It turns out that displacements (as functions of the design) generally do not converge for designs possessing zero densities in the limit. As a consequence, values depending on displacements (e.g., stresses) may be meaningless for structures with small densities. Moreover, the theoretical background of standard optimization solvers (e.g., first order necessary optimality conditions) breaks down. This lack is the reason for frequently observed misbehavior (see, e.g., [3]) and even failure when standard optimization solvers are used. Even worse, erroneous results may be calculated.

First we make some investigations on the relation between design and state variables for limiting designs with zero densities (see also [2]). We present new results showing that, under certain conditions, limiting displacements exist depending on the convergence speed of the design. Based on these investigations, optimality conditions for the design problem are investigated. Since we want to avoid setting zero lower bounds for the design variables, we consider classical problem formulations in design and displacement variables, i.e., formulations in a SAND (simultaneous analysis and design) approach. Meanwhile it is well understood that standard regularity conditions (so-called constraint qualifications) generally are not satisfied at local optimizers (see, e.g., [1]). The behavior of displacements, however, leads to results showing that necessary optimality conditions of the SAND formulation can be proved without the consideration of standard constraint qualifications.

The results in this talk are based on joint work with Christoph Schürhoff.

REFERENCES

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