Fast Construction of Matching Constraints for Three-Dimensional Domain Decomposition Methods with Non-Matching Grids

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ABSTRACT

Numerical methods relying on Domain Decomposition (DD) allow great flexibility in the discretization of partial differential equations since different types of fomulations and meshes can be adopted independently in any subdomain of the partition [1]. Moreover, DD methods naturally allows for parallel computing and multiphysics problems [2]-[3]. In this framework, the construction of matching conditions between subdomains becomes of pivotal importance. A novel coupling strategy, based on bi-orthogonal basis functions, for a fast construction of the mortar projection operator between non-conforming meshed domains was presented in [4].

In this work, starting from this approach, a novel mortar method for three-dimensional elliptic problems is proposed. The computational domain is subdivided into master and slave regions matching on an interface, which inherits its mesh from the slave side. A steady-state electric conduction problem is taken as an example. The scalar potential distribution in bulk domains is discretized by the Cell Method, where equations are expressed directly in algebraic form unlike FEM. Matching constraints at the interface are imposed by introducing an additional set of variables, i.e. Lagrange multipliers. With novel dual basis functions for multipliers [5], the projection operator mapping degrees of freedom from the master to the slave side can be constructed with minimum computing effort. The operator assembly is carried out in linear time by Gander's algorithm [6], when finding the intersection between the master and slave interface meshes.

Numerical results show that the proposed procedure is both applicable and reliable. The accuracy of matching conditions and the convergence behaviour for different mesh sizes will be presented.

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