3D Numerical Approximation of Relativistic Particle Beams

by Asymptotic Expansions

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ABSTRACT

Charged particle beams and plasma physics problems are extensively used in Science and Technology. Although we often associate accelerators with the large machines of high-energy physics, charged particle beams have continually expanding applications in many branches of research and technology. Recent active areas include flat-screen cathode-ray tubes, synchrotron light sources, beam lithography for microcircuits, thin-film technology, production of short-lived medical isotopes, radiation processing of food, and free-electron lasers. Clearly, there exists a significant interest in building mathematical models for these beams.

If we consider collisionless plasma or non-collisional beams, one of the most complete mathematical models is the time-dependent Vlasov-Maxwell system of equations. However, the numerical solution of such models requires a large computational effort. Therefore, whenever possible, we have to take into account the particularities of the physical problem to derive asymptotic approximate models leading to cheaper simulations.

In this talk, we consider the case of high energy short beams. A typical example is the transport of a bunch of highly relativistic charged particles in the interior of a perfectly conducting hollow tube. Numerical simulations are mostly performed using the particle-in-cell (PIC) method.

We propose a new paraxial model that approximates the coupled time-dependent Vlasov-Maxwell equations. Following [1], [2] the model is derived by introducing a frame which moves along the optical axis at the speed of light, so that the bunch of particles is evolving slowly in this frame. Then, one considers a scaling of the equations which reflects the characteristics of the high energy short beam. This allows us to introduce a small parameter η which denotes the ratio between the transverse characteristic velocity of the beam and the speed-of-light. We finally use asymptotic expansion techniques to obtain a new paraxial model which is accurate up to fourth order in η. The simplicity of the obtained formulation allows to use a finite-difference or finite element discretization for the Maxwell equations. Hence, using a particle approximation for the Vlasov equation, a particle-in-cell technique can be easily developed. This approach promises to be very powerful in its ability to get an accurate and fast algorithm.

REFERENCES

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