The Stokes Problem with Slip Boundary Conditions

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ABSTRACT

Observing a fluid flow along a solid impermeable wall, one can notice in some applications a variable tangential velocity of the fluid that may depend on a material quality or a shape of the wall. Such behaviour of the fluid is usually simulated by the slip boundary condition. It is used for modelling the blood flow, the metal forming processes, the polymer flow, or the hydrodynamics problems; see [1, 2] and references therein. Conditions of this type are used also in contact problems of solid mechanics, where they describe friction laws between bodies [3].

The research project deals with the Stokes flow with the threshold slip boundary conditions. A finite element approximation of the problem leads to the minimization of a non-differentiable energy functional subject to two linear equality constraints: the impermeability condition on the slip part of the boundary and the incompressibility of the fluid.

Eliminating the velocity components, one gets the smooth dual functional in terms of three Lagrange multipliers. The first Lagrange multiplier regularizes the problem. Its components are subject to simple bounds. The other two Lagrange multipliers treat the impermeability and the incompressibility conditions. The last Lagrange multiplier represents the pressure in the whole domain. The solution to the dual problem is computed by an active set strategy [4] and a path-following variant of the interior-point method [5]. To increase computational performance we propose the parallel implementation using the TFETI domain decomposition method [6]. Selected numerical experiments illustrate computational efficiency of the algorithms [7].

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