Decoupling Navier-Stokes / St. Venant-Kirchhoff Fluid-Structure Interaction by Optimization

Paul A. Kuberry*, Hyesuk K. Lee+
Clemson University
Clemson, SC 29630 USA
* Email: pkuberr@g.clemson.edu, Webpage: http://www.paulkuberry.com
+ Email: hklee@clemson.edu, Webpage: http://www.math.clemson.edu/~hklee

ABSTRACT

An approach is presented for simulating time-dependent fluid-structure interaction (FSI) between a Newtonian fluid modeled by the Navier-Stokes equations and a nonlinear elastic modeled by the St. Venant-Kirchhoff equations. A Neumann control is introduced into the strongly coupled FSI system. With the introduction of this control, the fluid and structure subsystems are now only weakly coupled through the fluid domain movement which is an extension of the structure velocity. The result is an optimization problem in which the traction force density function imposed on the interface is used as a control, with the goal of minimizing the difference in velocities on the interface. This decoupling allows for the use of partitioned solvers for the fluid and structure subsystems.

Although many options exist as to which optimization algorithm to use, numerical results will be given using the Gauss-Newton method as the outer optimization algorithm and the conjugate-gradient algorithm as the inner optimization algorithm. Due to the fast convergence of the Gauss-Newton method, generally only 3-4 outer optimization iterations occur at each time step. In each of these iterations, the update for the domain displacement and velocity must be calculated and correspondingly the fluid and structure matrices must be reassembled. In the inner optimization algorithm, however, only the adjoint and the Fréchet derivative of the state operators must be repeatedly solved. Since the domains are not updated within this inner optimization loop and the problems being solved are linear, the matrices corresponding to the fluid and structure operators can be factorized and the factorization can be reused within that iteration of the Gauss-Newton algorithm.

There are two negative properties to solving FSI by partitioned algorithms. Staggered algorithms such as the Dirichlet-Neumann coupling tend to be unstable or require relaxation which dramatically increases the number of state solves necessary. With regard to stability, since the traction force control is applied to both subsystems simultaneously, the instabilities seen due to the added-mass effect are not a factor in our approach. Because the solves required by Dirichlet-Neumann coupling are all nonlinear, the cost is great and reassembly of the matrices must occur at each iteration. Our approach avoids this problem, since few outer iterations of the optimization algorithm are needed. The result is that we can solve a fully coupled FSI problem in a small constant multiple of the time needed simply to perform the forward solves of the respective FSI subsystems.