Convergence speed of coupling iterations for the unsteady transmission problem

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ABSTRACT

Cooling of gas-turbine blades, thermal anti-icing systems of airplanes or supersonic re-entry of vehicles from space are examples of thermal fluid structure interaction. In particular gas quenching recently received a lot of industrial and scientific interest because, in contrast to liquid quenching, this process has the advantage of minimal environmental impact due to non-toxic quenching media and clean products like air.

Unsteady thermal fluid structure interaction is modelled using two partial differential equations describing a fluid and a structure on different domains. The equations are connected at an interface and they are coupled to model the heat transfer between fluid and structure.

We introduce here the transmission problem because it is a basic building block in fluid structure interaction. We consider the coupling of two heat equations on two identical squared domains. The Laplacian is discretized by second order central finite differences and the implicit Euler method is used for the time discretization. The coupling conditions are formulated on the interface. The standard algorithm to find solutions of the coupled problem is the Dirichlet-Neumann iteration, where the PDEs are solved separately using Dirichlet-, respectively Neumann boundary conditions with data given from the solution of the other problem as [1] describes. Convergence analysis for the Dirichlet-Neumann iteration can be found in [3].

In order to study the convergence behaviour of this coupled problem, we reformulate the fully-discretized equations as a system of algebraic equations. Henshaw and Chad provided in 2009 [2] a method to analyse stability and convergence speed for the thermal transmission problem based on applying the continuous Fourier transform to the semi-discretized equations. Their convergence criteria only depends on the thermal conductivities and diffusivities of the materials. Our numerical results for the fully-discretized case are not completely covered by this analysis, and therefore, we propose a complementary stability and convergence study for the fully-discretized equations that allows to study the differences between the semi-discretized and the fully-discretized cases. Numerical results are presented to illustrate the analysis.

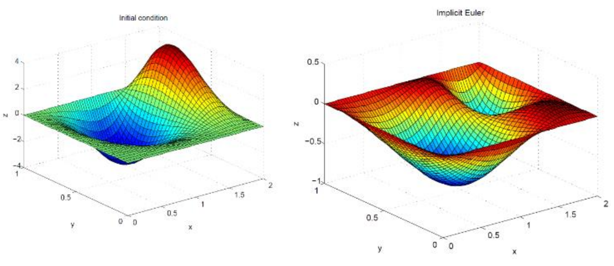

Figure 1: The initial condition and the solution after few time steps of the unsteady transmission problem. The two identical squared domains are described by [0,1] x [0,1] and [1,2] x [0,1].

REFERENCES

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