

Thermodynamically Consistent Phase Field Models for Two-phase Flows and Their Numerical Methods

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ABSTRACT

We will present some recent work on phase-field models for two-phase fluids with variable densities. The Quasi-Incompressible Navier-Stokes-Cahn-Hilliard model with the gravitational force being incorporated in the thermodynamically consistent framework will be investigated. Under a minor reformulation of the system we show that there is a continuous energy law underlying the system. For the reformulated system we then design a continuous finite element method and a special temporal scheme such that the energy law is accurately preserved at the discrete level. Such a discrete energy law for a variable density two-phase flow model has never been established before with continuous finite element. We will also show an example that an energy law preserving method will perform better for multiphase flow problems. Finally, we will develop a model to account for the thermocapillary effects. It allows for the different properties (densities, viscosities and heat conductivities) of each component while maintaining thermodynamic consistency. To our knowledge such a model is new. Numerical validation is provided too. The talk is based on two recent papers.

REFERENCES

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