ANALYSIS OF TIME FILTERS USED WITH THE LEAPFROG SCHEME

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ABSTRACT

The second order accurate leapfrog (LF) time-stepping scheme is widely used in the atmospheric numerical weather predictions (NWP). To alleviate the undamped computational mode it is usual to apply the Robert–Asselin (RA) time filter. Unfortunately, the RA time filter also weakly suppresses the physical mode, and it degrades the numerical accuracy to first order. In spite of these limitations, the RA-filtered LF scheme is widely used in various NWP models, due to its easy to implementation in legacy codes and and low computational cost.

Paul Williams recently proposed a modification to the RA filter to restore the second-order accuracy when used in conjunction with LF scheme, now known as the Robert–Asselin–Williams (RAW) filter. He also proposed two other time filters for the composite-tendency LF scheme. These two schemes are second-order accurate, and they can increase the accuracy for the amplitudes to fifth and seventh order, respectively.

In 2014, Li and Trenchea suggested a higher-order Robert–Asselin (hoRA) type time filter. When combined with LF scheme (LF-hoRA), the method is third order accurate. The non-intrusive LF-hoRA is almost as accurate, stable and efficient as the intrusive third-order Adams-Bashforth method.

A long-standing problem in the integration of NWP models is that the largest allowable time step is governed by the considerations of stability rather than accuracy. The atmosphere and ocean support several types of waves that propagate at very different speeds. If an explicit time differencing is used to approximate the time derivative in the equations governing a system, the maximum stable time step will be limited by the speed of the most rapidly propagating waves (sound waves and gravity-inertia waves). Early NWP models used the explicit LF scheme, and the maximum permissible time step has to be small enough for the integration to be stable. In order to relieve the severe time step restriction, it is common to treat the high-frequency waves implicitly and the remaining terms explicitly. This *semi-implicit* numerical technique was pioneered by Robert in 1969. In contrast to a fully explicit scheme, the semi-implicit scheme eliminates the time step restriction imposed by the high-frequency modes. In atmospheric sciences, the Crank-Nicolson-Leapfrog (CNLF) method is widely used. It treats the rapidly-moving waves implicitly using the Crank-Nicolson scheme to stabilize the fast-moving waves, while the remaining slower-moving waves are treated by the explicit leapfrog scheme.

In this talk we describe the behavior of the hoRA filter in the CNLF integrations (CNLF-hoRA) and compare it with CNLF-RAW,AI2*-AB3, BI2*-BX3*, AM2*-AX2* and BDF2-BX2*.

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