A numerical scheme for coupling gas dynamics and electrodynamics applied for fully and partially ionized high speed flows

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ABSTRASCT

The flow of ionized gases under the influence of electromagnetic fields is governed by the coupled system of the compressible flow equations and the Maxwell equations. In this system, coupling of the flow with the electromagnetic field is obtained through nonlinear and stiff source terms, which cause difficulties with the numerical solution of the coupled system. For fully ionized gases, a uniform electrical conductivity σ_e could be defined and coupling of the fluid dynamical equations with the Maxwell equations is achieved through the addition of Lorentz force, $\mathbf{J}\times\mathbf{B}$, and work, $\mathbf{J}\cdot(\mathbf{B}\times\mathbf{v})$, in the right hand side of the momentum and energy equations, respectively. In the absence of free charges and for uniform conductivity the magnetic induction vector $\mathbf{B}$ is related with electric field vector $\mathbf{E}$ thought Ohm's constitutive relation, $\mathbf{J}=\sigma_e(\mathbf{E}+\mathbf{v}\times\mathbf{B})$, where $\mathbf{v}$ is the velocity of the conducting gas. Under these assumptions full coupling of the equations governing flow and electromagnetic fields is obtained and a more extended than the MHD realization for ionized gas flows is obtained since the time variation of the electric displacement in the Maxwell equations is not ignored.

For partially ionized gases, on the other hand additional mass conservation equations must be used to evaluate the mass density of each species, such as electrons, ions and atoms, in the mixture. Furthermore, the coupled system is completed with a separate energy equation for electrons. The mass conservation equations of species include reaction and diffusion rates that depend on diffusion velocities. The diffusion velocities are from the kinetic theory of gases e.g. the Boltzmann's equation with the Chapman-Enskog collision integral. For partially ionized gas mixtures due to the presence of free charge, ρ_c, the Lorenz force per unit volume is $\mathbf{F}_L=\rho_c\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)+\mathbf{J}\times\mathbf{B}$. In the energy balance equation for the mixture the heat flux vector is accordingly modified to include contributions from electrons and energy transfer from heavy particles due to chemical reactions. In addition a separate equation for electrons is included, while for thermal plasmas (two temperature ionized gases) one more equation for positive ions and atoms must be included.

For both cases fully or partially ionized plasma flows, the discontinuous Galerkin finite element method is used for the numerical solution of the system coupling flow and electromagnetic fields. For the magnetic field vector, discontinuous Galerkin discretization is performed using a divergence-free vector base for the magnetic field to preserve zero divergence in the element and retain the implicit constraint of a divergence-free magnetic field vector down to very low level both globally and locally. To circumvent difficulties resulting from the presence of the stiff source terms, implicit time marching is used for the fully coupled system to avoid wrong wave shapes and propagation speeds that are obtained when the coupling source terms are lagged in time or by using splitting iterative schemes.

For fully ionized gas flow, numerical solutions for benchmark problems were computed on collocated meshes for the flow and electromagnetic field variables. The fully coupled monolithic approach showed good agreement with other numerical solutions and exact results for the MHD limit. Numerical solutions for Riemann problems obtained for partially ionized gases approach asymptotically the fully-ionized and non-ionized limit and demonstrated the expected behavior for different degrees of ionization. Furthermore, the present approach is demonstrated for the control of the shock standoff distance with use of strong magnetic field interacting with the high speed partially ionized gas flow.