Tolerance Modelling of Thermoelastic Phenomena in Functionally Graded Laminates

J. Jędrysiak* and E. Pazera†

* Department of Structural Mechanics
Łódź University of Technology
al. Politechniki 6, 90-924 Łódź, Poland
e-mail: jarek@p.lodz.pl

† Department of Structural Mechanics
Łódź University of Technology
al. Politechniki 6, 90-924 Łódź, Poland
Email: ewelinapazera@interia.pl

ABSTRACT

The problem of thermoelasticity in a two-phase laminate, with non-periodic distribution of the ingredients, is considered. The cells with constant thickness l are composed of two sublayers of different materials. The macroscopic properties change continuously in a direction perpendicular to laminas. Therefore, we deal here with *laminates with functionally graded properties*, [3]. In the analysis of various issues concerning these laminates, the same approaches are often used as for composites with periodic structures, cf. the book [3]. Unfortunately, most of the proposed equation models do not take into account *the effect of the microstructure size*.

In order to obtain the averaged equations, taking into account this effect, *the tolerance averaging technique* is applied, cf. [4], to describe various thermomechanical problems in periodic and non-periodic composites. Using the tolerance modelling the system of differential equations with slowly-varying coefficients is obtained, instead of highly oscillating, tolerance-periodic and discontinuous coefficients. For this purpose, the concept of a slowly-varying function, a tolerance-periodic function and an averaging operation are used, cf. [4,1].

Two basic assumptions are used in the modelling. The first of them is *the micro-macro decomposition*, where it is assumed that the fundamental unknowns (e.g - the displacement field and the temperature field) are taken as a sum of a averaged part (field) and an oscillating part. Furthermore, it is assumed that the oscillating part can be expressed as a product of the known *fluctuation shape function*, and the new unknown - *the fluctuation amplitude*. The new basic unknowns, averaged fields and fluctuation amplitudes, are assumed to be slowly-varying functions of that coordinate which parameterizes the perpendicular direction to laminas. The second assumption is *the tolerance averaging approximation*, in which it is assumed that some terms are negligible small.

The main aim of this note is to obtain and apply averaged differential equations of the thermoelasticity problems of considered laminates with transversally graded properties. Substituting the micro-macro decomposition to the main equations of thermoelasticity problems, [1,2], by doing the appropriate averaging and transformations, so the final equations of *the tolerance model* are derived, cf. [2].

REFERENCES

[1] J. Jędrysiak, *Thermomechanics of laminates, plates and shells with functionally graded properties*, Łódź, Wyd. PŁ, 2010, (in polish).

[2] J. Jędrysiak, "On the tolerance modelling of thermoelasticity problems for transversally graded laminates", *Arch. Civ. Mech. Eng.*, vol. **11**, pp. 61-74, (2011).

[3] S. Suresh and A. Mortensen, *Fundamentals of functionally graded materials*, Cambridge, The University Press, 1998.

[4] C. Woźniak, B. Michalak and J. Jędrysiak, (eds.), *Thermomechanics of microheterogeneous solids and structures*, Łódź, Wyd. PŁ, 2008.