Efficient Discretization of Multiwave Systems

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ABSTRACT

Numerical methods for wave propagation problems typically leverage the finite domain-of-dependence of the solution operator to utilize efficient explicit time-stepping schemes, often based on standard linear multistep or Runge-Kutta formulas. However, for systems with multiple wave families or problems in multiple media it is possible that both fast and slow waves will be present [1]. Basic examples include compressible flows at low Mach number [2] or the shallow water equations [3]. Various methods have been proposed to treat such problems, but in general the use of standard time-stepping schemes will require either small steps dictated by the fast waves or implicit solves with highly nonsymmetric matrices. In this talk we consider a different approach based on stable, explicit, integral-based time stepping formulas. Examples of the use of such formulas to derive unconditionally stable explicit time-stepping methods for the scalar wave equation [4-6] and linear acoustics [7-8] have appeared. Here we develop extensions of these methods to account for wave generation by coupling with possibly nonlinear slow waves.

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