Formulation, analysis and computation of an optimization-based local-to-nonlocal coupling method

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ABSTRACT

Nonlocal models are used in many scientific and engineering applications where the material dynamics depends on the microstructure. This work presents an optimization-based method for the coupling of local and nonlocal continuum models [1]; the coupling aims to combine the computational efficiency of partial differential equations with the accuracy of nonlocal models. The need for local-to-nonlocal couplings is especially acute when the size of the computational domain is such that the nonlocal solution becomes prohibitively expensive to compute, yet the nonlocal model is required to accurately resolve small scale features such as crack tips or dislocations that can affect the global material behavior.

We formulate the coupling as a control problem where the states are the solutions of the nonlocal and local equations, the objective is to minimize their mismatch on the overlap of the local and nonlocal domains, and the controls are the nonlocal volume constraint and the local boundary condition. We provide a mathematical and numerical analysis of the method in the context of nonlocal diffusion models [2]. Numerical examples in one-dimension illustrate the theoretical properties of the approach and provide the basis for realistic simulations.

REFERENCES

[1] M. D'Elia, P. Bochev, “Formulation, Analysis And Computation of an Optimization–Based Local–To–Nonlocal Coupling Method”, submitted, 2014.

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