Coupling Requirements for Well Posed and Stable Multi-physics Problems

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ABSTRACT

Roughly speaking, a well posed initial boundary value problem require that a unique solution that can be estimated in terms of the data exist. The most common procedure for showing well posedness is by the use of the so called energy-method where one multiply the governing partial differential equations (PDE's) by the solution, integrate by parts and impose boundary conditions [1].

The same knowledge is not wide-spread when it comes to mathematical coupling of multi-physics problems. The reason for that is mainly due to the more complex and to some extent more unclear nature of a coupling procedures compared to imposing boundary conditions.

Firstly, accuracy relations must exist such that combinations of variables for one set of PDE's at the interface is equal to combinations of variables for the other set. Secondly, the number of accuracy relations must fit both problems. Too many conditions ruin existence and too few ruin uniqueness. Thirdly, the accuracy relations must lead to a dissipative coupling.

We will investigate the problems mentioned above and generalize the investigation in [2] where we deived the standard fluid-structure coupling by only demanding a well posed problem. Combinations of hyperbolic PDE's where the number of coupling conditions vary with the sign of the eigenvalues of the matrix in the quadratic form at the interface will be considered.

Once the coupling conditions are known for the continuous multi-physics problem we will discretize using high order finite differences on summation-by-parts form and include the coupling conditions weakly using the SBP-SAT technique [3],[4],[5]. Stability will be proven and numerical experiments will be presented.

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