XVIII International Conference on Water Resources
CMWR 2010
J. Carrera (Ed)
CIMNE, Barcelona 2010

METHOD OF LEAST SQUARES FOR COMPUTING NORMALS AND CURVATURES FROM 2D VOLUME FRACTIONS

Poorya A. Ferdowsi* and Markus Bussmann†

Department of Mechanical and industrial engineering, University of Toronto,
5 King's College Rd., Toronto, Ontario, Canada
* e-mail: p.ferdowsi@utoronto.ca
† e-mail: bussmann@mie.utoronto.ca, web page: http://www.mie.utoronto.ca/faculty/bussmann

Summary: Among numerous methods for numerical simulation of two-phase flows, e.g. wave impact to marine vessels and ocean structures, the volume-of-fluid (VoF) method is one that can exactly preserve the volume of each phase [4, 5]. In the VoF method, the volume fraction f of one phase is considered as a reference, and defined in each cell as the ratio of the volume occupied by that phase to the total cell volume. Cells with $f = 1$ and $f = 0$ are then full of one or the other fluid; those cells with $0 < f < 1$ define the interface between the phases. Such a volume fraction field is discontinuous, and so it is difficult to obtain accurate estimates of interface normals and curvatures, required both by the geometric advection algorithms that are typical of VoF methods, and for the surface tension term that must be calculated by an accompanying flow solver [2]. Here, we present a new method for calculating normals and curvatures from volume fractions.
Many methods have been developed for computing normals [3, 4, 5] among which the height function (HF) method has been used extensively due to its simplicity and accuracy [1]. Other, more complicated, methods fit curves to the volume fraction data and then evaluate normals and curvatures from the curves, but such methods are complex and expensive because they obtain the curve fits iteratively to minimize some objective function (e.g. LVIRA).
A Hybrid HF / Arc Fit Least-square Technique: A more complex way to compute normals and curvature is to use a fitting curve like an arc, and combine the least-square and HF concepts. This method uses the concept of an osculating circle which is the circle that best approximates the curve at a point. The discretized error function is minimized iteratively subject to a constraint to obtain the best fit. When the arc is obtained, the normal at the midpoint of the arc segment confined by the cell is considered the interface normal, and the reciprocal of the arc radius is the curvature of the interface. This method obtains normals and curvature for a line and a circle up to machine precision, and it is more accurately than any other method we are aware of, particularly in under-resolved regions.

References

[1] S. J. Cummins, M. M. Francois, and D. B. Kothe. Estimating curvature from volume fractions. Computers and Structures, 83(6-7):425–434, 2005.
[2] M. M. Francois, S. J. Cummins, E. D. Dendy, D. B. Kothe, J. M. Sicilian, and M. W. Williams. A balanced-force algorithm for continuous and sharp interfacial surface tension models within a volume tracking framework. Journal of Computational Physics, 213(1):141–

173, 2006.
[3] J. E. Pilliod Jr. and E. G. Puckett. Second-order accurate volume-of-fluid algorithms for tracking material interfaces. Journal of Computational Physics, 199(2):465–502, 2004.
[4] W. J. Rider and D. B. Kothe. Reconstructing volume tracking. Journal of Computational Physics, 141(2):112–152, 1998.
[5] R. Scardovelli and S. Zaleski. Direct numerical simulation of free-surface and interfacial flow. Annual Review of Fluid Mechanics, 31:567–603, 1999.