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EXPERIMENTAL AND NUMERICAL SIMULATION OF GRAVITY CURRENTS ON SLOPING BEDS

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Summary. The aim of this paper is the investigation of gravity currents moving on beds with different slopes by both laboratory experiments and numerical simulations. Laboratory experiments are performed by full-depth lock exchange release and an image analysis technique is applied to measure the space-time evolution of the gravity current's profile. A two-layer, 1D, shallow-water model is used to simulate gravity currents. The model takes into account the space-time evolution of free-surface and the mixing due to the gravity current. Entrainment at the interface between the gravity current and the ambient fluid is modeled by a modified Ellison & Turner's formula (1959)[1]. Several tests are run to calibrate an entrainment parameter in order to reproduce gravity currents moving both on a horizontal and upsloping beds. The comparison between numerical and experimental results shows that the developed model is a valid tool to reproduce gravity current's dynamics.

1 INTRODUCTION

Gravity currents occur when a fluid flows into another fluid of different density. These phenomena are very common both in natural and industrial flows. Avalanches, pyroslastic flows, sea breeze wind are gravity currents driven by the difference in bouyancy between the dense and the ambient fluid. This density gradient can be due to a difference of temperature (as the case of sea breeze wind), salinity (as the case of the Mediterranean outflow) or to the presence of suspended sediments (i.e. turbidity currents). An exhaustive set of examples can be found in Simpson (1997)[2]. Several investigations studied the dynamics of gravity currents by both numerical and experimental analysis. Most of the models simulating gravity currents are based on the shallow water approximation as Rottman & Simpson (1983)[3], Shin et al. (2004)[4], La Rocca et al. (2008)[5] and Adduce et al. (2009)[6]. When a gravity current moves, it mixes with the ambient fluid. A recent investigation on the parametrization of mixing due to gravity currents is given by Cenedese and Adduce (2008)[7].

In this paper experimental gravity currents were produced by lock exchange release experiments, which were conducted in a Perspex tank of rectangular cross-section divided into two parts, one filled with tap water and the other one filled with salt water, separated by a sliding gate, as shown in Figure 1. The experiment begins when the sliding gate is suddenly removed and the heavier fluid moves from the left part of the tank to the right part forming a gravity current. The experiment stops when the front of the gravity current reaches the right wall of the tank.

The dynamics of gravity currents, obtained by an instantaneous release on a flat bed, can be divided into three distinct phases (Simpson, (1997)[2]; Marino et al., (2005)[8]). During the first phase, called slumping phase, the front position varies linearly with time and the front speed is constant. During the second phase or self-similar phase the front speed U_f varies with $t^{-1/3}$ (i.e. the front position depends on time by a law $t^{2/3}$). The transition to the second phase occurs when a wave, generated by the reflection of the lighter fluid to the left wall, reaches the current's front, which is slower than the wave. The third phase or viscous phase occurs if viscous effects become predominant. During the third phase the front speed U_f decreases with $t^{-4/5}$ (i.e. the front position depends on time by a law $t^{1/5}$).

The aim of this paper is the investigation of gravity currents moving on beds with different slopes by both laboratory experiments and numerical simulations. Most of the recent works concerns about down-sloping gravity currents. One of the innovative aspect of this paper is to focus about gravity currents on upsloping bed with free surface.Four laboratory experiments were performed by a lock release with four different bed's slopes. A two-layer, 1D, shallow-water model was used to simulate gravity currents. The mathematical model takes into account the space-time evolution of the free surface, which allows a better agreement between the measured and the simulated gravity currents. The model also accounts for the mixing between the two fluids, which causes a decrease of gravity current's density. The mixing between the two layers was modeled by a modified Ellison and Turner's (1959)[1] formula. Several simulations were run varying a calibration parameter of the model to investigate the relation between entrainment and gravity current's velocity.

2 EXPERIMENTAL SET UP

The experiments were performed at the Hydraulics Laboratory of University Roma Tre. Gravity currents were generated in a tank of rectangular cross-section, 3.00 m long, 0.30 m deep, 0.20 m wide and with transparent Perspex sidewalls. A sketch of the tank is shown in Figure 1. The tank was divided into two parts by a sliding vertical gate placed at a distance x_0 from the left end wall. The right part of the tank was filled with fresh water of density ρ_2, while the rest of the tank was filled with salty water with initial density $\rho_{01} > \rho_2$. Both in the right and in the left part of the tank the depth of the fluid was h_0. A pycnometer was used to performe density measurements. A quantity of dye was dissolved into the salt water to provide the flow visualization during the experiment. The experiment starts when the sliding gate is suddenly removed and the heavier fluid moves from the left part of the tank to the right part forming a gravity current. The experiment stops when the front of the gravity current reaches the right wall of the tank. A CCD camera, with a frequency of 25 Hz, was used to record the experiments and an image analysis technique, based on a threshold method, was applied to measure the gravity currents' profiles. Each pixel of the acquired images is characterized by a number ranging from 0 (black) to 255 (white). The grey level of the interface between the two fluids was choosen as the threshold value. The program travelled along the columns of the image until it met the threshold value (i.e. the interface between the two fluids) and recorded the coordinates of this pixel as a point of the current's profile. A rule was positioned along both the horizontal and vertical walls of the tank in order to obtain the conversion factor pixel/cm.

Four experiments were performed keeping constant ρ_{01}, ρ_2, h_0, x_0 and varying the bed's slope ϑ. The desired slope was obtained by placing the tank above a structure and changing its inclination. Four values of ϑ were investigated: 0°, 1.39°, 1.45° and 1.8°. ϑ = 1.45° is the critical angle for ρ_{01} = 1090 kg/m^3, i.e. the gravity current reaches the left end wall of the tank with a front's velocity close to zero. For Run 2 the gravity current reaches the end wall with a velocity higher than zero (i.e. subcritical slope), while for Run 4 the gravity current doesn't reach the end of the channel (i.e. supercritical slope). The experimental parameters are shown in Table1.

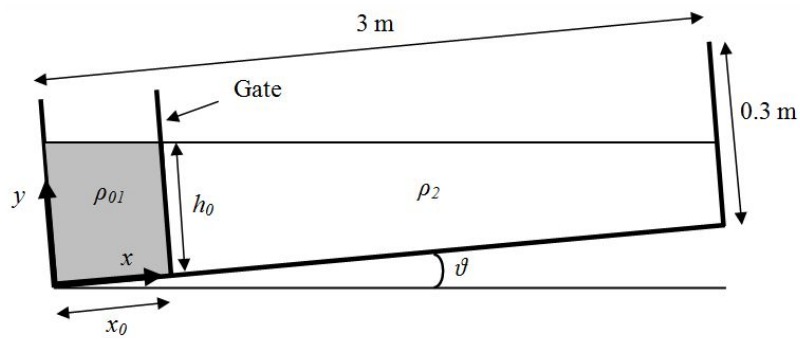

Figure 1: Definition sketch of the tank used for the experiments

Run	x_0 [m]	h_0 [m]	ρ_{01} [Kg/m^3]	ρ_2 [Kg/m^3]	ϑ [°]
1	0.1	0.15	1090	1000	0
2	0.1	0.15	1090	1000	1.39
3	0.1	0.15	1090	1000	1.45
4	0.1	0.15	1090	1000	1.8

Table 1: Experimental parameters

3 MATHEMATICAL MODEL

A two-layer, 1D, shallow-water model was used to simulate gravity currents. Gravity currents frequently delevop along the longitudinal direction, so that the ratio between the depth and the lenght of the current is small enough to allow the application of the shallow water theory. Several authors investigated gravity currents by shallow water equations (Rottman & Simspon (1983)[3], Sparks et al. (1993)[9], Hogg et al. (1999)[10]).

These authors assumed a steady free surface, while in the present work this hypotesis has been removed in order to have a more realistic solution, modelling the space-time evolution of the free surface. The mathematical model takes also into account the mixing between the two fluids. The entrainment at the interface, due to a mass transport from the lighter fluid to the heavier one, causes a decrease of the density of the gravity current. The entrainment between the two fluids was modeled by a modified Ellison and Turner's formula (1959)[1]. Figure 2 shows the frame of reference used in the model.

A monodimensional gravity current moving on a bed of a slope ϑ is considered. For the mathematical model, negative values of ϑ are referred to upsloping beds. The heavier current of height h_1 and density ρ_1 flows below the ligther one of height h_2 and density ρ_2. Applying principle of mass conservation and projecting along the axis x the momentum equations, the following system of hyperbolic partial differential equations is obtained:

$$\begin{cases} \dfrac{\partial(\rho_1 h_1)}{\partial t} + \dfrac{\partial(\rho_1 V_1 h_1)}{\partial x} = \rho_2 V_e \\ \dfrac{\partial(\rho_2 h_2)}{\partial t} + \dfrac{\partial(\rho_2 V_2 h_2)}{\partial x} = -\rho_2 V_e \\ \dfrac{\partial V_1}{\partial t} = g \sin\theta - \dfrac{\partial}{\partial x}\left(\dfrac{\rho_1 h_1 + \rho_2 h_2}{\rho_1} g \sin\theta + \dfrac{V_1^2}{2}\right) - \dfrac{\tau_{1b} + \tau_{12}}{\rho_1 h_1} \\ \dfrac{\partial V_2}{\partial t} = g \sin\theta - \dfrac{\partial}{\partial x}\left((h_1 + h_2) g \sin\theta + \dfrac{V_2^2}{2}\right) + \dfrac{\tau_{12} + \tau_{2b}}{\rho_2 h_2} \end{cases} \tag{1}$$

where the unknown quantities h_1, h_2, V_1 and V_2 are the depth and the velocity of the lower and the upper layer, respectively, V_e is the entrainment velocity, τ_{1b} and τ_{2b} are the stress terms between the two fluids and the bottom (these terms include both bed's stress and lateral walls stress), and τ_{12} is the stress at the interface between the two fluids. The bottom stress is modelled by Darcy-Weisbach's formula (1857, 1845)[11-12]:

$$\tau_{ib} = \lambda_i \rho_i \frac{V_i |V_i|}{8} \tag{2}$$

where λ_i, the friction factor, was defined by Colebrook (1939)[13]:

$$\lambda_i = \lambda_{i\infty}\left(1 + \frac{8 h_i}{\mathrm{Re}_i\, \varepsilon}\right) \tag{3}$$

where $\lambda_{i\infty}$, Re_i and ε/h_i are the friction factor for turbulent rough flows, the Reynolds number and the relative roughness of the i_{th} layer, respectively. $\lambda_{i\infty}$ and Re are defined as:

$$\lambda_{i\infty} = \frac{1}{4}\left[\log\left(\frac{3.71 h_i}{\mathrm{Re}_i\, \varepsilon}\right)\right]^{-2} \tag{4}$$

$$\mathrm{Re}_i = \frac{V_i D_i}{\nu} \tag{5}$$

Equation (3) shows that the term $\frac{8h_i}{Re_i\,\varepsilon}$ adapt the friction factor for turbulent rough flows to turbulent transition flows. In the performed experiments turbulent transition flows develop.

The stress at the interface between the two fluids is defined as:

$$\tau_{12} = \lambda_{12}\frac{\rho_1+\rho_2}{2}\frac{(V_2-V_1)|V_2-V_1|}{8} \tag{6}$$

In this study several runs were performed to calibrate the friction factor at the interface λ_{12}. The value λ_{12} = 0.24 was found as the optimum value and it was used for all the simulations. A modified Ellison & Turner's formula (1959)[1] was used to model the entrainment. V_e is given as a function of the Froude number of the gravity current, defined as:

$$Fr = \frac{V_1}{\sqrt{h_1\frac{\rho_1-\rho_2}{\rho_1}g\cos\theta}} \tag{7}$$

Because Ellison & Turner's formula was obtained by an experimental apparatus different from the lock exchange experiment, in this paper some modifications to Ellison & Turner's relation were adopted. The relation used in this paper to model the entrainment parameter is:

$$\frac{V_e}{|V_1|} = \frac{k\cdot Fr^2}{Fr^2+5} \tag{8}$$

where k is a dimensionless coefficient to be calibrated. The entrainment velocity increases as k increases. The optimum k has to balance a correct evaluation of the gravity current's depth and a good simulation of the front's speed of the gravity current. The mathematical model was numerically solved by an esplicit Mac-Cormack' finite difference scheme by predictor-corrector scheme.

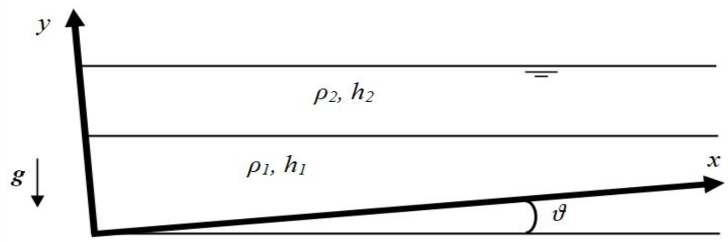

Figure 2: Frame of reference used in the mathematical model

4 COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

Different tests were performed with different values of the dimensionless parameter k, in order to obtain the optimum value The calibration of k is necessary because equation (8) doesn't take into account that entrainment depends also on the Reynolds number of the gravity current (Cenedese & Adduce, 2008)[7] and this is an empirical formula which is a first attempt to model the entrainment in the shallow water framework. As a consequence we found two different optimum values of k: for the numerical simulation of Run 1 (i.e. performed on horizontal bed) k = 0.6 was the optimum value, while k =1.1 was found as optimum value for Runs 2, 3, 4 (i.e. performed on upsloping beds).

Figure 4a-d shows the comparisons of numerical gravity current's profiles and the images acquired by the camera for Run 1, at three different time steps after release, t=8 s (Figure 4a), t=12 s (Figure 4b), t=18 s (Figure 4c) and t=24 s (Figure 4d), respectively: solid line represents the current's profile for miscible fluids with k=0.6, while dotted lines show current's profiles for immiscible fluids (i.e. k=0). A simulation without mixing causes a reduction of the height of the gravity current, as shown in Figure 4a-c. The effect of mixing is to produce a mass flow from the lighter fluid to the heavier one, causing an increase of the height of the current's profile and therefore a decrease of the density of the gravity current.

Figure 5 shows the space-time evolution of Run 1 obtained by laboratory measurement, numerical simulation for miscible fluids (i.e. k=0.6) and immiscible fluids (i.e. k=0). This comparison shows that if mixing is not taken into account the numerical prediction is in agreement with experimental measurement just for the initial stage of gravity current's development. The numerical simulation obtained with k=0.6 shows a good agreement with the measured front position.

Figure 6 shows a comparison between numerical and experimental front position versus time for all the performed runs. The numerical results shown in Figure 6 are obtained for miscible fluids, using the optimum value for k. All the laboratory measurements of the current's profile started about 2 second after the gate removal, because it was difficult to measure the profile of the currents during this initial stage.

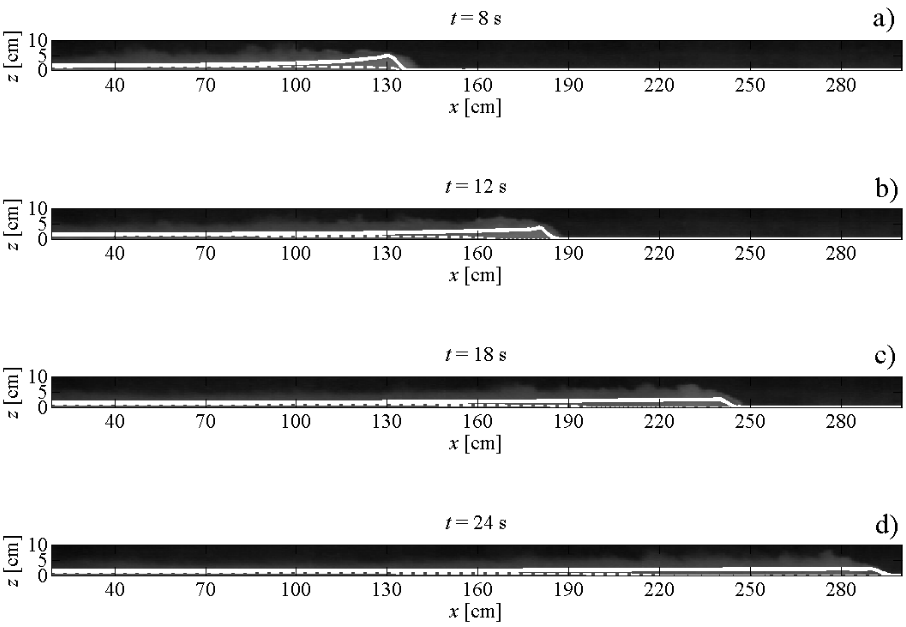

Figure 4a-d: Comparison of numerical gravity current's profiles for Run 1 and the images acquired by the camera at four different time steps: miscible fluid, k=0.6 (solid line) and immiscible fluid, k=0 (dotted line).

The comparison shows a good agreement between experimental currents' front position and the numerical predictions. Figure 6 shows that gravity current's velocity decrease as the slope increases, then Runs 1, 2, 3 reach the end of the slope, while Run 4 stops at about 2.43 m.

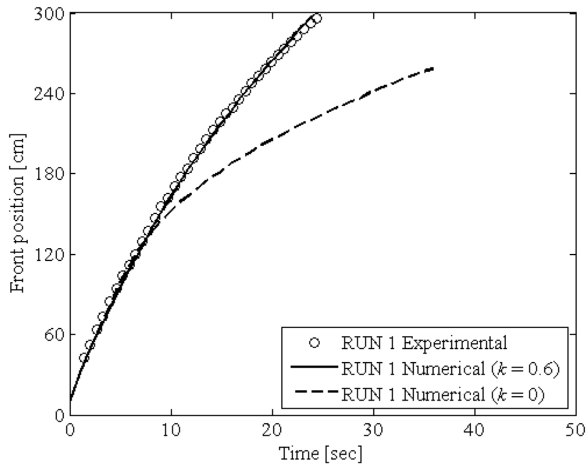

Figure 5: Front position versus time for Run 1: measurements (circles), numerical simulation with k=0.6 (solid line) and numerical simulation with k=0 (dotted line).

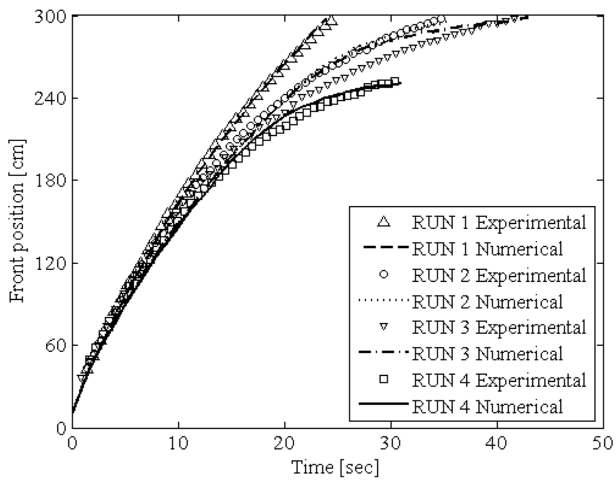

Figure 6: Experimental and numerical front positions versus time for all the runs

5 CONCLUSIONS

In this work experimental and numerical simulations of gravity currents on sloping beds were performed. Four full-depth lock exchange release experiments were realized to compare laboratory results to numerical simulations keeping costant the initial density of gravity current, the initial position of the vertical gate and the initial depth of the two fluids, and changing the bed's slope. All the experiments were recorded by a camera and the space-time evolution of the gravity currents was measured by an image analysis technique. Numerical simulations were run by a two-layer, 1D, shallow-water model. The oscillation of the free surface and the mixing between the two layers are taken into account in the mathematical model. The entrainment at the interface, due to a mass transport from the lighter fluid to the heavier one, causes a decrease of the density of the gravity current. The entrainment between the two fluids was modeled by a modified Ellison & Turner's formula (1959)[1]. Several tests were run to calibrate the adimensional coefficient k in order to have a good simulation of both the front position and the profile of the gravity current. The comparison between experimental measurements and numerical results for miscible fluids shows a good agreement both for the gravity current's profiles and for the front's positions. Therefore the presented model is able to reproduce gravity current's dynamics both on horizontal beds and on upsloping beds.

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