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LONGITUDINAL DISPERSION UNDER WIND-GENERATED WAVES

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Summary. *The waves-current-turbulence interactions affect significantly the dynamics of the surface layer of lakes and oceans and are thus of particular importance for predicting transfers through the top meters of water. The aim of the paper is to evaluate the mean effect of these interactions on the dispersion of marked fluid particles in the wind direction. The complete set of equations governing the wind drift current and the Langmuir circulations is first solved numerically. The structure of the turbulent flow is calculated using a k-ε turbulent model. The influence of wave breaking is deduced from the available experimental cases and is parameterized as a surface source of turbulence. The computed vertical distributions of the longitudinal drift current and the eddy viscosity are then used to deduce the longitudinal dispersion coefficient, D. Compared with there established in the usual shear flows, obtained Formula shows first that the longitudinal spreading of marked fluid is augmented by the supplement of turbulence induced by wave breaking and, second, that the Langmuir circulations are lateral irregularities which reduce the longitudinal dispersion. These results are therefore in good agreement with the qualitative indications found in literature.*

1 INTRODUCTION

In estuaries, lakes, and oceans, the interactions between wind-generated waves, wind-induced mean current, and turbulence induce a particular organization of the flow, very different from the configuration of the usual shear flows and for which two prominent features are well observed: the appearance, under some conditions, of Langmuir circulations and the high level of turbulent intensities measured below the wind waves. Langmuir circulations consist of a quasi-steady pattern of parallel vortices, usually oriented downwind, and of alternating sense of circulation or vorticity. Observations of turbulence close to the air-water interface found that, under strong wind forcing, turbulent quantities are greater than wall-layer values.

The role of turbulence in promoting mixing is well known[3,6,13,16]. In contrast, the Langmuir circulations effects are still poorly understood or quantified even their effects on vertical transport and diffusion have been described[10,11] and their possible importance in horizontal dispersion have been recognized[5]. The question of the combined action of Langmuir circulations and turbulence on the spreading of marked fluid particles is not easy to dress. However, as will be seen below, the computed vertical distributions of the longitudinal drift current and the eddy viscosity may be used to evaluate the mean effect of this combined action on the dispersion of marked fluid particles in the longitudinal wind direction.

2 CHARACTERISTICS OF THE DOMINANT GRAVITY WAVE

When wind blows over a water surface, it exerts a stress at the interface thereby inducing in the water a sheared turbulent drift current, and creates a pattern of random gravity surface waves. Many field or in laboratory experimental studies[15,18] shows that energy spectrum exhibited a peak at the dominant frequency. The wave pattern is therefore usually represented by the dominant gravity wave characterized by its amplitude a, wave number K and frequency σ. Many attempts have been made to relate these parameters to the external wind forcing[8,14]. However, since the growth of waves involves different processes like propagation over sheared flow, breaking, turbulence and waves-waves interactions, most of these investigations are unable to explain observations of wave evolution over the whole range of wind and wave speeds. The established relations for the wave parameters (a, K, σ) takes or not into account the state of wave development.

The wave's generation mechanisms are associated to the energy flux transmitted from the wind to the water by normal stresses on the air-water interface. This energy flux can be normalized by u_{*g}^2, where u_{*g} is the friction velocity in the gas. The wave evolution depends upon the fetch X. The appropriate physical variables for describing the dominant gravity wave evolution are then u_{*g}^2, X and g, where the gravitational acceleration g is intended to account for the presence of gravity waves at the interface. Assuming that the air and water densities are constant, that the water is of infinite depth (then the dispersion relation $\sigma^2 = gK$ is valuable) and using dimensional analysis, the main features of the dominant gravity wave, expressed in dimensionless forms as $K^* = Ku_{*g}^2 / g$ and $a^* = ga / u_{*g}^2$, should be functions of $X^* = gX / u_{*g}^2$. Using results of experimental studies carried out in lake Ontario[9,17] and in laboratory tanks[1,12,18], the following correlations are obtained[2] :

$$K^* = 18.7 X^{*-0.6} \tag{1}$$

$$a^* = 0.02 X^{*0.5} \tag{2}$$

Figure 1 shows that, independently of conditions under which experimental measurements are carried (field or laboratory facilities, wind speeds, breaking ...), the established relations are in good agreement with the used experimental points. Presuming that breaking or wave-current interactions are responsible for the high level of turbulence measured below the interface, there must be a surface layer whose scaling is controlled by various characteristics of the wave field, such as wave height and the state of wave development. As seen before, the wave parameters are well correlated to X* and the friction velocity, the fetch and the gravitational acceleration are the more meaningful physical parameters for describing the scaling of the upper-layer turbulence.

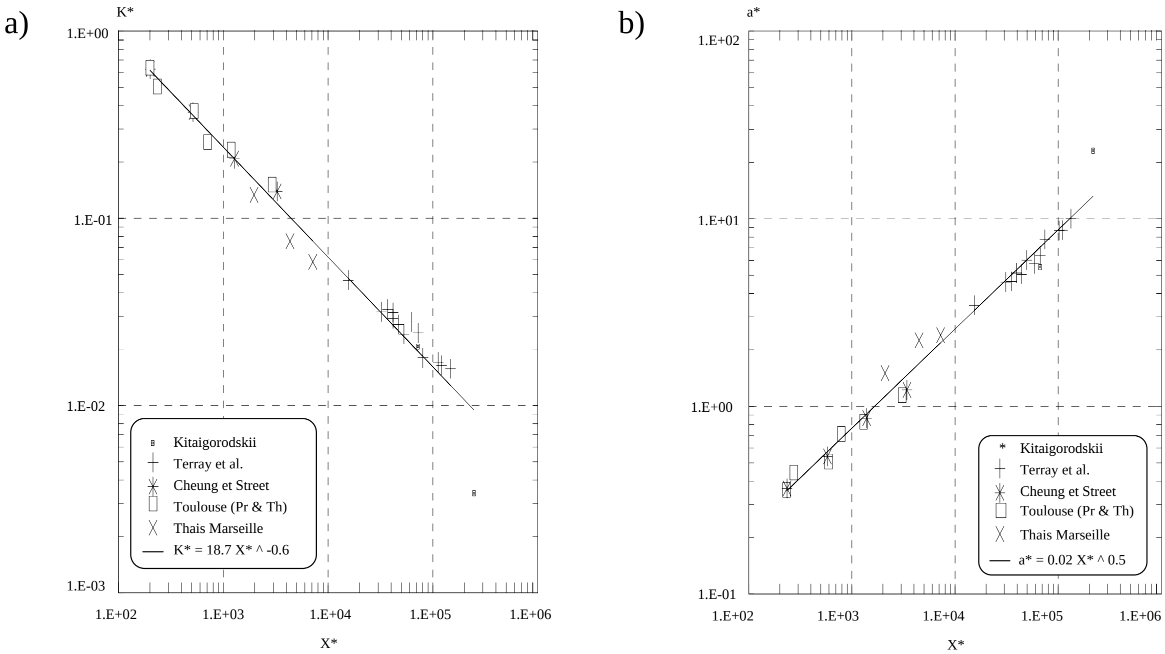

Figure 1 : Wave parameters (K*,a*) versus fetch (X*).

3 UPPER-LAYER TURBULENCE SCALING

Since all characteristics of wind-generated waves seems well correlated to non dimensional fetch X*, we suppose that turbulence scales beneath the interface are also correlated to X*. The turbulent kinetic energy k_i, normalized by u_{*w}^2, with u_{*w} the friction velocity in the water at the interface, and the kinetic energy dissipation rate ε_i, normalized by gu_{*w}, are functions of X*. Using extrapolation to the near-surface layer of turbulence measurements carried out in Lake Ontario[9,17] and in laboratory tanks[1,12,18], the following correlations are established[2]:

$$k_i / u_{*w}^2 = 4.3X^{*0.2} \tag{3}$$

$$\varepsilon_i / gu_{*w} = 0.6X^{*-0.5} \tag{4}$$

where subscript "i" refer to mean air-water interface (the top of the computational domain).

4 MODEL FORMULATION

The main target of the numerical model for wind drift current, Langmuir circulations and turbulent quantities under wind waves is the calculation of the mean current which develop in response to the wind stress ($\tau_w = \rho u_{*w}^2$, with ρ the water density) imposed on the mean air-water interface (z = 0) and to the wave-induced driving force (left through the Stokes drift U_s). We deal with the problem in two space dimensions (y, z plane), assuming that variations in the longitudinal wind direction are negligible ($\partial / \partial x = 0$). This assumption has strong justification from observations[11]. Since the mean cellular motion is assumed periodic in the cross-wind direction, the computational domain is limited to half of the y period motion that comprises one Langmuir cell. The correspondent rectangular domain is delimited by a solid wall at the bottom (z = -h), by the mean air-water interface at the top (z = 0), and by the right and left sides (y = 0, l), where l is half of the spacing between windrows.

The finite element discretization scheme in space used in the developed model is the well known Galerkin weighted residual finite element method using quadratic elements. The time dependant terms are approximated using linear elements. The proposed model for simulation is used in parallel flow and with Langmuir circulations for the available experimental cases. Details of the model solved equations, numerical procedure and computed features of the drift current, Langmuir circulations and turbulence intensities are not described in this paper. We only note that, compared with the corresponding field measurements, the results indicate that the space variations of the velocity field and the turbulent intensities are predicted satisfactory[2]. Here we will focus on the averaged results over two neighbors' counter-rotating cells. The averaged vertical distributions of the longitudinal drift current and the eddy viscosity are in fact the only which will be used to estimate the longitudinal dispersion coefficient.

6 HORIZONTALLY AVERAGED RESULTS

The solution for wind drift current, Langmuir circulations and turbulent quantities is either parallel or a periodic function of y. In the last case, after the steady solution has been obtained, its average over one period in y (i.e. two neighbors' counter-rotating cells) is computed to yield the horizontally averaged solution. By the symmetry of the problem, the only non-zero mean velocity component of this average is in the wind direction.

The normalized wind drifts and eddy viscosities vertical profiles are plotted in figure 2 for some experimental cases with different values of the parameter X*. Calculations have been carried in parallel flow and with Langmuir Circulations, when these are present. Compared to the corresponding parallel flow solution, Langmuir circulations reduce the vertical gradient of the drift velocity. For high secondary-flow components intensities, the effect of mixing in cross-wind section on the drift profile is pronounced. The eddy viscosity distributions are also subject to the convective action of Langmuir cells. The turbulent behavior in the near-surface is mainly controlled by the specified boundary conditions for k and ε and the effect of Langmuir cells is particularly apparent in the central region. The numerical solution also depends upon the geometrical dimensions of the computational domain and the water current. Nevertheless, deviations between computed profiles plotted on figure 2 show that X* is the main control parameter of the wind drift current, Langmuir circulations and turbulence solution under wind waves.

Since the horizontally averaged secondary-flow components in the y and z directions are zero, the averaged mean motion is "parallel" and, in a tentative way, the well known Taylor's analytical method can be extended to estimate the longitudinal dispersion coefficient under wind-generated waves.

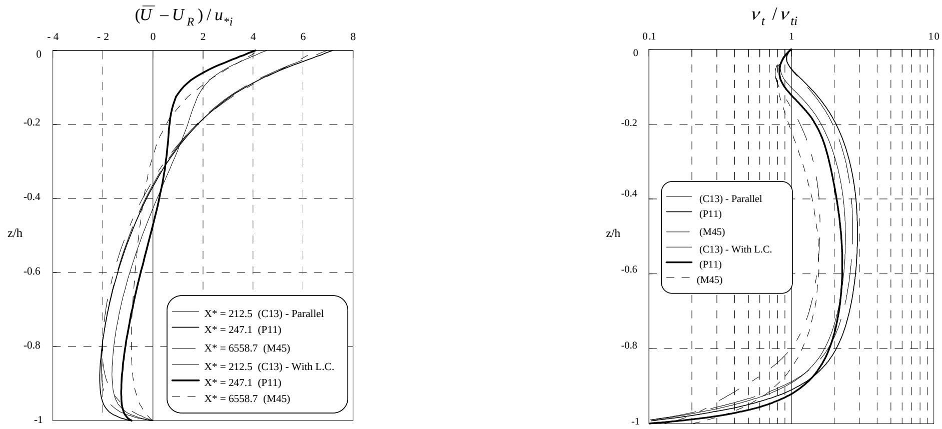

Figure 2 : Horizontally averaged velocity and eddy viscosity distributions.

7 LONGITUDINAL DISPERSION COEFFICIENT

We consider in the following only passive dispersants (i.e. those that are miscible and do not alter the assumed constant density or the velocity of the fluid). The mean concentration of the tracer $\overline{c}(x, y, z, t)$ can be written as consisting of a cross-sectional mean value $<\overline{c}>$ and a deviation thereform c'. For long time enough, longitudinal dispersion is quite usually assumed Fickian. The rate at which mass is transported through a cross-section moving at the water current is $M = -AD(\partial <\overline{c}> / \partial x)$, where A is the cross-sectional area and D is the longitudinal dispersion coefficient (or coefficient of apparent diffusivity in the x direction) of the cross-sectional mean concentration $(<\overline{c}>)$ as used in the one-dimensional equation

$$\frac{\partial <\overline{c}>}{\partial t} + <\overline{u}> \frac{\partial <\overline{c}>}{\partial x} = D \frac{\partial^2 <\overline{c}>}{\partial x^2} \tag{5}$$

$$D = -\frac{1}{A} \int_{-h}^{0} u' \left(\int_{-h}^{z} \frac{\sigma_c}{\nu_t} \left(\int_{-h}^{z} u' dz \right) dz \right) dz \tag{6}$$

where A is the cross section, $<\overline{u}>$ is the water current (the cross-sectional mean value of $\overline{u}$), $u' = \overline{u} - <\overline{u}>$, σ_c is the turbulent Prandtl-Schmidt number.

Expression (6) shows that the spreading of marked fluid particles in the mean motion direction depend upon the combined action of the velocity and the eddy diffusivity. Elder[3] applied Taylor's asymptotic analysis to a two-dimensional open-channel flow and evaluate D using Von-Kármán's vertical velocity profile $u' = (u_{*p} / \kappa)(1 + \ln(-z/h))$, where h is the water depth, u_{*p} the bottom shear velocity, and κ the Von-Kármán's constant. The eddy diffusivity was obtained assuming Reynold's analogy ($\sigma_c = 1.0$) and using the linear vertical momentum transfer which gives $\nu_t = (-z/h)(1 + z/h)\kappa h u_{*p}$. Putting u' and ν_t expressions into expression (6) gives $D = (0.404/\kappa^3) h u_{*p}$ and then $D = 5.9 h u_{*p}$ for κ = 0.41.

7.1 Parallel flow case

When there are surface gravity waves, the mean motion is either parallel or including secondary-flow components in the cross-wind section. In the first case, a strong wind forcing reveals higher turbulent intensities than expected from purely shear-driven wall layer. For each available experimental case, the proposed numerical model was first used to simulate the parallel flow expected under the prescribed external forcing. The obtained eddy viscosity and velocity distributions were then introduced in expression (6) and a numerical estimation of the longitudinal dispersion coefficient was made. The obtained values, normalized by hu_{*i}, are then dressed versus the dimensionless fetch X*. As shown on figure 3a, the dimensionless longitudinal dispersion coefficient, D/hu_{*i}, seams to be quite well correlated to the parameter X* and the best obtained relation is :

$$\frac{D}{hu_{*i}} = 6.65 \times X^{*0.027} \tag{7}$$

Using purely shear-driven wall layer hypothesis, the normalized longitudinal dispersion coefficient was found approximately equal to 7.6. Under wind waves, this value is reached at X* = 140 and then very near the wind inlet, there where the production-dissipation equilibrium of turbulence may be applied. Using the available experiments, the obtained result shows also that the normalized longitudinal dispersion coefficient varies from 7 to 10. The longitudinal spreading of marked fluid is then augmented by the supplement of turbulence induced by wave breaking.

7.2 Effect of Langmuir cells

The horizontally averaged eddy viscosity and drift velocity distributions constitute the mean effect of Langmuir cells on mean velocity and turbulence fields. In first approximation and in a same way as before, these distributions was used to estimate the effect of Langmuir circulations on the longitudinal spreading of tracer. Using the horizontally averaged longitudinal velocity $\overline{U}(z)$ and turbulent viscosity $\overline{\nu}_t(z)$ profiles obtained in the last section, expression (6) was numerically integrated to yield an approximate value of D/hu_{*i} for each simulated case. The obtained numerical estimations are dressed on figure 3b versus the parameter X*. The obtained relation $D/hu_{*i} = f(X^*)$ is

$$\frac{D}{hu_{*i}} = 0.64 \times X^{*0.28} \tag{8}$$

This relation, compared with there established for parallel flows, shows that the Langmuir circulations reduce the longitudinal dispersion coefficient. This result is in good agreement with the qualitative indications found in literature. Erdogan and Chatwin[4] and Fischer[6,7] assert that lateral irregularities reduce the longitudinal dispersion of tracers.

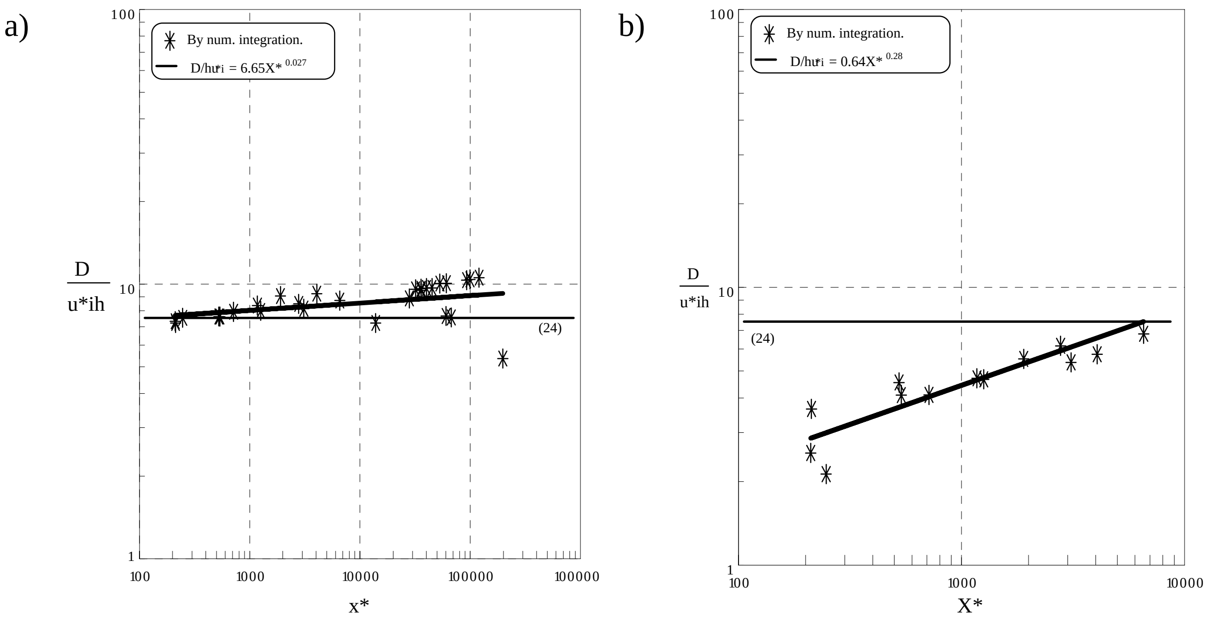

Figure 3 : Normalized longitudinal dispersion coefficient versus X*.
(a) Parallel flow; (b) With Langmuir Circulations

8 CONCLUSIONS

Our aim in this work was to evaluate the mean effect of waves-current-turbulence interactions on the dispersion of marked fluid particles in the longitudinal wind direction. The equations governing the wind drift current and the Langmuir circulations were first solved numerically using a k-ε turbulent model. In the developed model, the effect of wave breaking is parameterized as a surface source of turbulence. The parameterization is deduced from the available experimental cases and it's found closely related to the fetch, X, and to the friction velocity in the water, u_{*w}. Simulations are carried in parallel flow and with Langmuir circulations, when these are present. The results are compared with the corresponding field measurements and they indicate that the space variations of the velocity field and the turbulent quantities are predicted satisfactory. The computed vertical distributions of the longitudinal drift current and the eddy viscosity are then used to deduce the longitudinal dispersion coefficient, D, by use of the Taylor theory. In the two cases, with and without Langmuir circulations, the coefficient D is found closely related to the parameters (X, u_{*w}, h), where h is the water depth. Formulas, compared with there established in the usual shear flows, show first that the longitudinal spreading of marked fluid is augmented by the supplement of turbulence induced by wave breaking and, second, that the Langmuir circulations are lateral irregularities which reduce the longitudinal dispersion. These results are therefore in good agreement with the qualitative indications found in literature.

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