XVIII International Conference on Water Resources
CMWR 2010
J. Carrera (Ed)
CIMNE, Barcelona 2010

LABORATORY AND NUMERICAL INVESTIGATIONS OF VARIABLE DENSITY-FLOW AND TRANSPORT IN HELE-SHAW CELL

H. T. Trieu[*], C. Oltéan[*], F., Golfier[*] and M. A. Buès[*]

[*] Nancy Université – Laboratoire Environnement, Géomécanique & Ouvrages,
Ecole Nationale Supérieure de Géologie, Rue du Doyen Marcel Roubault,
BP 40, 54501, Vandœuvre-lès-Nancy, France

Key words: density driven-flow, salt solute infiltration, Hele-Shaw cell, numerical model

Summary. This paper deals with the experimental and numerical simulations of variable-density flow and solute transport in an analogous model of a homogeneous and saturated porous medium. The analogous model is constituted by a transparent Hele-Shaw cell. The numerical simulations were performed with the FRIPE code that uses a combination of the mixed hybrid finite element and discontinuous finite element methods. The specific point of interest in this study is the investigation of dense plume behavior locally injected into an “ambient groundwater flow field”.

1 INTRODUCTION

In case of accidental or natural groundwater contamination (e.g., saltwater intrusion, infiltration of leachates from industrial waste disposal sites or landfills, leakage from salt lake and saline disposal basin), the change in concentration can affect the density and/or viscosity of fluids brought in contact. These physical parameters may alter the aquifer hydrodynamic properties (permeability, velocity field) and hence the hydrodynamic dispersion that constitutes the main factor in the development and spreading of the mixing zone. In order to better understand the influence of these factors on plume propagation, several experimental and numerical studies were carried out (e.g., Schincariol and Schwartz[1]; Hassanizadeh and Leijnse[2]; Simmons et al.[3]; Welty et al.[4]; Johansen et al.[5]; Nick et al.[6]).

Despite the attention that this type of problem has received, very few experimental works (e.g., Schincariol and Schwartz[1]; Oostrom et al.[7], Oltéan et al.[8]) have taken into account the density effects on a pollutant infiltration from a localized and continuous source into an ambient groundwater (flow perpendicular to gravity). Moreover, we must note that, in these laboratory studies, the experimental data are presented either in a qualitative way (Schincariol and Schwartz[1]) either in a quantitative way but in some singular points only located inside the mixing zone (Oostrom et al.[7], Oltéan et al.[8]). In order to mitigate these aspects and to: (i) visualize and measure the spatial concentration distribution in any point of the mixing zone and (ii) minimize the local permeability perturbations, we used a laboratory experimental model represented by a transparent Hele-Shaw cell. Considered as an analogous model of a homogeneous porous medium, this cell enabled us to employ a non-intrusive technique based on the Laser Induced Fluorescence (LIF) method. The experimental data were then used to verify: (i) the numerical solutions obtained with the computer code FRIPE (Buès and Oltéan[9]) and (ii) the stability criterion recently established by Musuuza et al.[10].

2 DENSITY DRIVEN-FLOW AND TRANSPORT IN POROUS MEDIA

The displacement of a non-reactive solute into a saturated porous medium is generally governed by a system of equations constituted by Darcy's law, the fluid and solute mass balances (Bear[11]). This set of partial differential equations is coupled by two state equations. In the mixing zone, the fluid density ρ [M/L^3] and the fluid dynamic viscosity μ [M/LT] are expressed as:

$$\rho = \rho_0\left(1+\alpha\overline{C}_m\right) \quad \text{and} \quad \mu = \mu_0\left(1+\beta\overline{C}_m\right) \tag{1}$$

where $\overline{C}_m$ is the reduced solute mass fraction [$C_m/C_{m,max}$], ρ_0 and μ_0 are, respectively, the ambient density and dynamic viscosity. α [-] and β [-] are experimental constants (Weast[12]).

In order to ensure (e.g., Brezzi and Fortin[13]; Chavent and Roberts[14]; Siegel et al.[15]; Ackerer et al.[16]; Buès and Oltéan[9]): (i) the local mass conservation and flux continuity, (ii) to capture the steep concentration front with a minimum spatial discretization and (iii) to minimize the diffusion phenomenon and numerical oscillations, the system is solved by using the mixed hybrid finite element method (MHFEM) and discontinuous finite element method (DFEM). The first one is applied to Darcy’s law, continuity equation and the dispersive term of the transport equation. The second one, coupled with a slope-limiting technique (Gowda and Jaffré[17]), is used to solve the advective term of the solute mass balance equation. These numerical techniques were implemented into a density driven simulator called FRIPE (Fluide Réacifs ou Inertes en Poreux hEterogènes).

The originality of this development consists in the choice, in the mass balance equation, of vector q which has the properties of Raviart-Thomas space. So, if this vector is identified as q = ρ.*V*, where *V* [L/T] is the fluid velocity vector, then the so-called “conservative form” (i.e., mass flux continuity) of the flow equation is preserved (Buès and Oltéan[9]). Moreover, this formulation does not introduce the density approximation on element edges (Ackerer et al.[16]) and therefore the matrix obtained by hybridation remains symmetrical and positive definite. The resulting system is solved in a sequential way using a standard fixed-point (Picard) scheme.

3 EXPERIMENTAL SET-UP

3.1 Experimental apparatus and methodology

The physical model, represented by a Hele-Shaw cell, is similar with that used by Oltéan et al.[18]. It is made by two parallel transparent plates disposed vertically and placed in an aluminum enclosure which is used to set the flow channel width between the plates (figure 1). Note that this aperture fixes the intrinsic permeability of the analogous porous medium. The plates are made of optical glass 300 mm in height, 200 mm wide and 10 mm thick. A needle placed inside the cell was used for the mass flux injection (inner diameter = 0.25 mm, outer diameter = 0.46 mm). In order to assure a constant mass flux, the needle was connected to a step by step motorized push syringe. As our study relates the behavior of a confined groundwater system with ambient flow, the upper and lower boundaries of the cell were made watertight while the lateral boundaries were connected to outflow constant head tanks. The constant velocity field inside the flow channel is assured by a particular dispositive placed between the two tanks. A system of immersed masses driven by a motorized wheel is used to maintain a constant hydraulic gradient between the two tanks without any fluctuation.

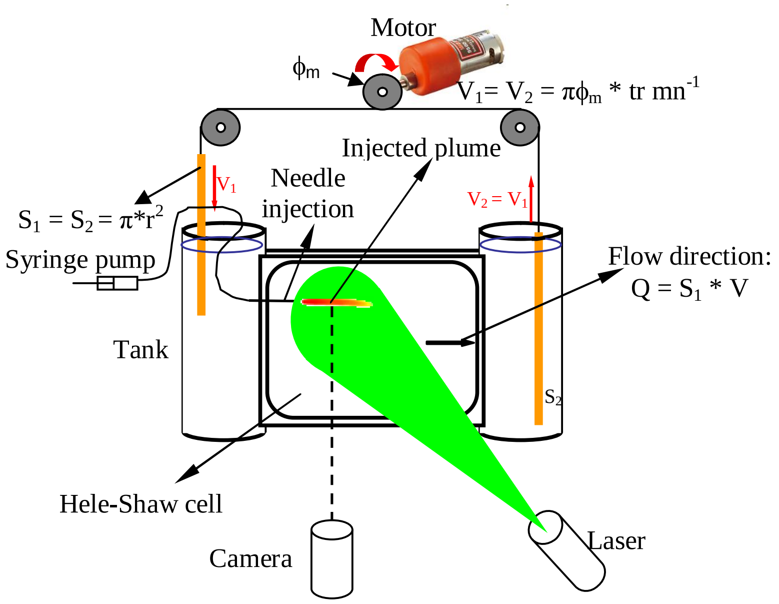


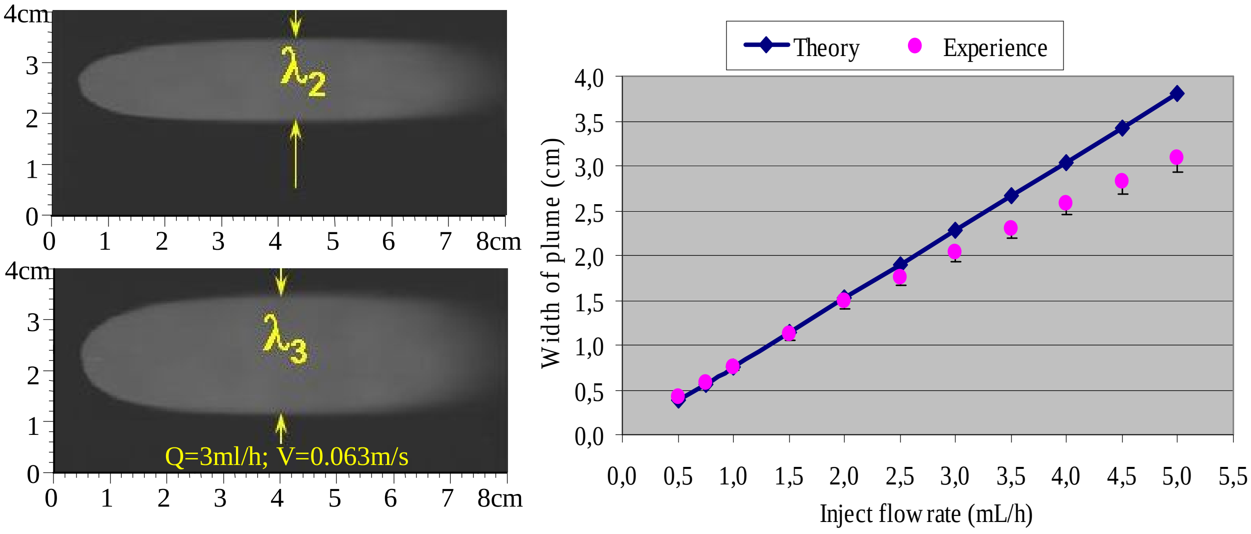
Figure 1. Schematic view of the experimental device

As measurement method we used the LIF (Laser Induced Fluorescence) technique. It is based on the excitation of a fluorescent dye (e.g., Rhodamine 6G) included in the studied fluid. In our case, the fluorescent dye, mixed with the pollutant (e.g., a salt solution), was excited by a double cavity pulsed laser (wavelength at 532 nm) and its displacement was followed and recording by a 12 bits definition digital camera. However, we must note that within experimental conditions presented here, this technique raises additional difficulties. In fact, as we cannot get the laser plane inside the Hele-Shaw cell, the laser plan is not any more perpendicular to the camera axis (figure 1).

This specific configuration requires the consideration of two aspects of the light energy repartition of the laser. The spatial energy repartition is unstable over time and the light energy across a laser beam is not constant but rather has a Gaussian repartition vs. radius. This means that not only the energy level of the spot is non uniform but it also varies from pulse to pulse. In order to overcome these effects, we used the same methodology as Mainhagu et al.[19] by taking into account a stack of consecutive images. So, the studied physical phenomenon was analyzed using an average image resulting from ten consecutive rough images. It is obvious that during the recording of the ten images (i.e., 4.5 seconds) we considered that the plume propagation is in a quasi steady state.

3.2 Testing the model validity

Firstly, we tested the velocity field using the PIV (Particle Image Velocimetry) method. So, without injection pollutant, we can note that the velocity field is perfectly horizontal and is in very good agreement with the theoretical value given by Darcy's law. Then, we tested the analogous conditions between Hele-Shaw cell and a homogeneous and isotropic porous medium. This test concerns the injection of a tracer into a uniform flow field already established in Hele-Shaw cell (Figure 2a).



(a) (b)

Figure 2. Experimental and analytical width of the tracer plume

From a theoretical point of view, this experiment can be assimilated to the well known configuration that is a source in a horizontal and uniform flow. The analytical solution leads to a finger-shape plume of width $\lambda = Q_v/Ub$, where Q_v $[L^3T^{-1}]$ represents the source flow rate, U $[LT^{-1}]$ the horizontal components of the velocity field and b [L] the thickness of the flow channel. This analytical solution was compared to the experimental one in the Figure 2b. The analysis of this figure highlights that, the plume experimental width is in very good agreement with the analytical solution, as long as the $Qv \leq 2$ mL/h. For $Q_v > 2$ mL/h the divergence in plume width could be due to 3D and/or inertial effects. Under these considerations, the analogy conditions between the two systems can be supposed as valid only for the injected flow rates $Qv \leq 2$ mL/h. This result is practically identical with that found by Oltéan et al.[20] which studied numerically the validity domain of the analogy conditions between these two systems for similar experiments but without lateral flow.

4 COMPARISION BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

4.1 Experimental results

The experimental trials presented in this paper permit to examine the influence of the injected flow rate on the spreading pattern of a dense plume. The injected concentration ranged from 0.1 to 0.5 g/L while the injected flow rate varies between 0.5 to 2.0 mL/h. In the Figure 3 we find some experimental results for an initial lateral flow velocity of 0.06 mm/s.

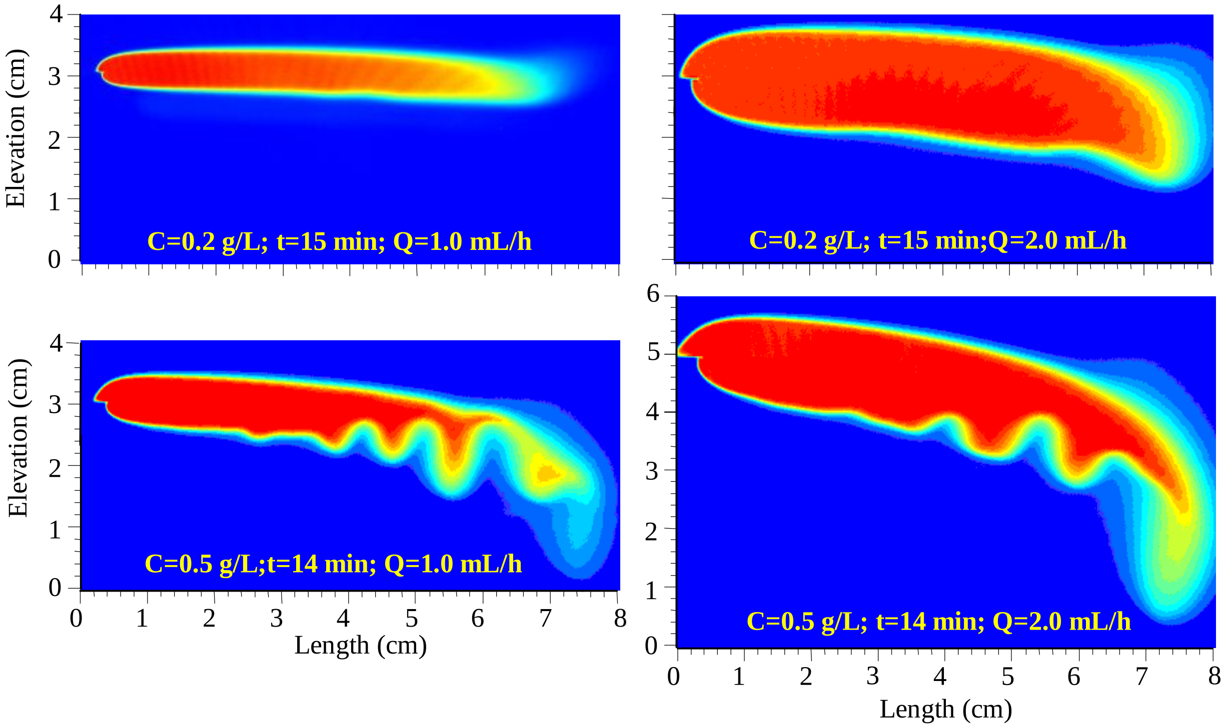


Figure 3. Experimental results

Contrary to Schincariol and Schwartz[1], the threshold for which the density effects can not be neglected in our case is about 0.5 g/L. For an injected concentration of about 0.2 g/L the plume remains more or less horizontal, with no apparent instabilities. On the other hand, for injected concentrations greater than 0.5 g/L we observe the formation and development of gravitational instabilities. Under these conditions, at least two questions remain: (i) can we use, for our experimental set-up, the stability criteria found in the literature in order to determine the threshold between the stable and unstable configurations? (ii) can we recover with our numerical code the experimental patterns observed? These issues will be addressed in the next section.

4.2 Numerical results

As mentioned in the first part of this paper, the experiments were simulated with FRIPE numerical code. In order to assure the analogy conditions between Hele-Shaw cell and porous media, the dispersion tensor was estimated using generalized Taylor dispersion tensor (Oltean et al.[20]). The studied domain as well as the initial and boundary conditions were identical with those imposed to experimental device. The porosity is set to 1. The other data taking into account in our simulations are summarized in table 1.

Parameters	**Value**	**Parameters**	**Value**
Water dynamic viscosity [Pa.s]	1.002e-3	Salt molecular diffusivity [m^2s^{-1}]	1.5e-9
Water ambient density [kg m^{-3}]	998.23	Intrinsic permeability [m^2]	2.52e-8
Injected salt concentration [kg m^{-3}]	0.2 and 0.5	Injected flow rate Q_V [mL/h^{-1}]	1.0 and 2.0

Table 1: Data for the numerical experiments

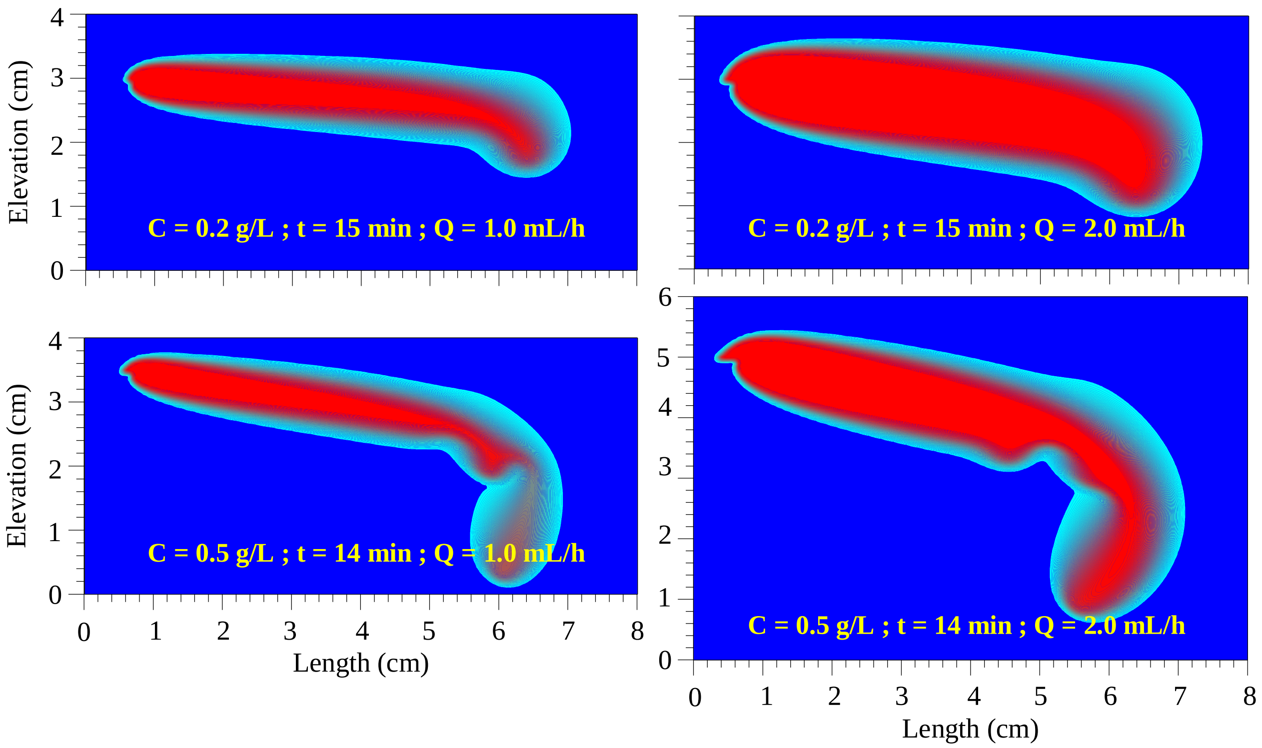


Figure 4. Numerical results

The numerical results are presented in the Figure 4. The qualitative analysis of the space-time evolutions of the mixing zone suggests a good agreement between experimental and numerical results. In spite of this, we can note some differences. For stable configurations, it appears that: (i) the numerical spreading of the mixing zone is slightly larger than that observed in the experimental data and (ii) the numerical plume is more tilted than the experimental one. Concerning the unstable configuration: (i) the number of lobes observed in the lower part of the numerical mixing zone is not the same with that observed for the experimental mixing zone and (ii) the longitudinal advance of the numerical mixing zone seems to be more reduced than that observed in the experimental data.

4.3 Verification of stability criteria

In this paragraph we purpose to verify one of the stability criteria recently proposed by Musuuza et al.[10]. Tested through numerical simulations based on the experimental results of Schincariol and Schwartz[1], it seems that this new stability criterion, expressed as:

$$\Lambda_0 = \frac{L}{D_{\|}}\left(v_0^g G_2 + v_0^p G_1\right)\left[(\alpha - \beta) + \left((a^2/(a^2+1))\right)\alpha\right] \qquad (2)$$

could predict the unstable ($\Lambda_0 < 0$)/stable ($\Lambda_0 > 0$) configuration of the mixing zone. In this last formula, α and β have the same significations than in equation (1), v_0^p the large scale pressure driven velocity component, v_0^g the gravity-driven velocity component, $a = v_0^g / v_0^p$, $D_{\|}$ the longitudinal dispersion, L the length of the simulated domain and G_1, G_2 the maximum concentration gradients in the orthogonal and parallel directions to gravity respectively.

As specified by Mussuza et al.[10], if the experimental/numerical data are characterized by a very small v_0^p, the equation (2) can be simplified and the flow remains stable as long as ($2\alpha - \beta$) < 0. By taking into account the α and β values (Weast[12]), it results that the flow should be unstable for injected concentrations higher than ≈ 2.5 g/L. As this conclusion is not in conformity with our experimental/numerical results, it appears that the stability number proposed by the authors mentioned above is not universally valid.

5 CONCLUSIONS

The development of an experimental model of Hele-Shaw type, enable us to analyze the space-time evolution of a salt solution injected into an “ambient groundwater flow field”. For our experimental conditions, a density difference as small as 0.5 g/L causes instabilities to appear in homogeneous medium. The experimental data were numerically analyzed using the FRIPE code. A good agreement was observed between experimental and numerical data. Concerning the stability analysis, it appears that “to predict unstable behavior is still a challenge in investigating density driven flows” (Mussuza et al.[10]).

REFERENCES

[1] R.A. Schincariol and F.W. Schwartz, “An experimental investigation of variable-density flow and mixing in homogeneous and heterogeneous media”, *Water Resour. Res*, **26** 2317-2329 (1990).

[2] S.M. Hassanizadeh and A. Leijnse, “A non-linear theory of high-concentration-gradient dispersion in porous media”, *Adv. Water Res*, **18**, 203-215 (1995).

[3] C.T. Simmons, Th.R. Fenstemarker and J.M. Sharp Jr., “Variable-density groundwater flow and solute transport in heterogeneous porous media: approaches, resolutions and future challenges”, *J. Contam. Hydrol*, **52**, 245-275 (2001).

[4] C. Welty, A.C. Kane and L.J. Kauffman, “Stochastic analysis of transverse dispersion in density-coupled transport in aquifers”, *Water Resour. Res*, **39**, SBH51-SBH518 (2003).

[5] K. Johannsen, S. Oswald, R. Held and W. Kinzelbach, “Coupled groundwater flow and transport: 1. Verification of variable density flow and transport models”, *Adv. Water Resour*, **29**, 1690-1704 (2006).

[6] H.M. Nick, R. Schotting, M. Gutierrez-Neri and K. Johannsen, “Modeling transverse dispersion and variable density flow in porous media”, *Transp. Porous Media*, **78**, 11-35 (2009).

[7] M. Oostrom, J.S. Hayworth, J.H. Dane and O. Güven, “Behaviour of dense aqueous phase leachate plumes in homogeneous porous media”, *Water Resour. Res*, **28**, 2123-2134 (1992).

[8] C. Oltéan, Ph. Ackerer and M.A. Buès, “Solute transport in 3D laboratory model through an homogeneous porous media: behaviour of dense aqueous phase and simulation”, *X International Conference Computational Methods in Water Resour, Heidelberg, Germany*, **1**, 341-346 (1994).

[9] M.A. Buès and C. Oltéan, “Numerical simulations for saltwater intrusion by the mixed hybrid finite element method and discontinuous finite element method”, *Transp. Porous Media*, **40**, 171-200 (2000).

[10] J-L. Musuuza, S. Attinger, F-A. Radu, “An extended stability criterion for density-driven flows in homogenous porous media”, *Adv. Water Res*, **32**, 796-808 (2009).

[11] J. Bear, *Hydraulics of Groundwater*, McGraw-Hill, New-York (1979).

[12] R.C. Weast, CRC Handbook of Chemistry and Physics, 58th ed., Chemical Rubber Company Press, Boca Raton (1977).

[13] F. Brezzi and M. Fortin, *Mixed and Hybrid Finite Element Methods*, Springer, Berlin, (1991).

[14] G. Chavent and J. Roberts, “A unified physical presentation of mixed, mixed-hybrid finite elements and standard finite difference approximations for the determination of velocities in water flow problems”, *Adv. Water Resour*, **14**, 329-348 (1991).

[15] P. Siegel, R. Mosé, Ph. Ackerer and J. Jaffré, “Solution of the advection dispersion equation using a combination of discontinuousand mixed finite elements”, *I. J. Num. Methods Fluids*, **24**, 595-613 (1997).

[16] Ph. Ackerer, A. Younes and R. Mosé, “Modeling variable density flow and solute transport in porous medium: Numerical model and verification”, *Transp. Porous Media,* **35**, 345-373 (1999).

[17] V. Gowda and J. Jaffré, *A discontinuous finite element method for scalar nonlinear conservation laws*, Rapport de recherche INRIA (1993).

[18] C. Oltéan, Ch. Felder, M. Panfilov and M.A. Buès, “Transport with a very low density contrast in Hele-Shaw cell and porous media: evolution of the mixing zone”, *Transp. Porous Media*, **55**, 339-360 (2004).

[19] J. Mainhagu, C. Oltéan, F. Golfier and M.A. Buès, “Measurement by laser induced fluorescence on miscible density driven flows in a Hele-Shaw cell: settings and preliminary results”, *C.R. Mécanique*, **335**, 105-112 (2007).

[20] C. Oltéan, F. Golfier and M.A. Buès, “Experimental and numerical study of the validity of Hele-Shaw cell as analogue model for variable-density flow in homogeneous porous media”, *Adv. Water Resour*, **31**, 82-95 (2008).