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GEOPHYSICAL INVERSION USING A HIERARCHICAL STRATEGY

Alex Furman[*] and Johan A. Huisman[†]

[*] Civil and Environmental Engineering, Technion, Haifa, Israel 32000
e-mail: afurman@technion.ac.il

[†] ICG 4: Agrosphere, Forschungszentrum Jülich, Germany
e-mail: s.huisman@fz-juelich.de

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Summary. Geophysical investigation is a powerful tool that allows non-invasive and nondestructive mapping of subsurface states and properties. However, non-uniqueness associated with the inversion process limits the quantitative use of these methods. One major direction researchers are going is constraining the inverse problem by hydrological observations and models. An alternative to the commonly used direct inversion methods are global optimization schemes (such as genetic algorithms and Monte Carlo Markov Chain methods). However, the major limitation here is the desired high resolution of the tomographic image, which leads to a large number of parameters and an unreasonably high computational effort when using global optimization schemes. One way to overcome these problems is to combine the advantages of both direct and global inversion methods through hierarchical inversion. That is, starting the inversion with relatively coarse resolution of parameters, achieving good inversion using one of the two inversion schemes (global or direct), and then refining the resolution and applying a combination of global and direct inversion schemes for the whole domain or locally. In this work, we explore the option of using a global optimization scheme for inversion of electrical resistivity tomography data through hierarchical refinement of the model resolution using a synthetic case study.

1 INTRODUCTION

Geophysical methods are increasingly used in hydrological and environmental applications and research. The benefits associated with geophysical investigations allow practitioners and researchers insight in the subsurface that was previously limited, invasive, destructive and expensive. Briefly, geophysical investigation includes data acquisition, inversion, and interpretation. The data acquisition stage includes collection of actively generated or passive geophysical signals, typically mechanic or electromagnetic. The inversion stage includes application of mathematical procedures to transform the obtained data to spatially distributed geophysical properties. The interpretation stage means translation of the obtained results to the desired understanding of the subsurface, e.g. identification of faults, spatial distribution of water content or solute concentration. To name a few examples from water-related fields with relevance to the specific geophysical method presented in this paper, Michot et al.[1] used electrical resistivity tomography (ERT) to map the water content dynamics in a corn field. Slater et al.[2] mapped biogenic gas in peat soil using ERT. Kemna et al.[3] used ERT to monitor the movement of a solute plume in groundwater.

Although geophysical investigation promises a significant benefit to subsurface research, the fact that it is not accurate enough is a limitation that makes it more of a qualitative tool at this stage. This inaccuracy is primarily due to the geophysical inverse problem being ill-posed. In other words, identification of many parameters using relatively little, somewhat noisy, information that is focused at the domain boundary is not perfect. Attempts to constrain the geophysical inverse problem can reduce the inversion error, yet are also limited. For example, Huisman et al.[4] used a Bayesian coupled inversion approach to determine the hydraulic properties of a river dike. In such a coupled inversion, an unsaturated flow model describing water infiltration into the dike is used to constrain the inversion of the ERT data. Bamberger et al.[5] also used unsaturated flow models to constrain ERT inversion for detection of water content distribution around an infiltration front. Nevertheless, the inversion accuracy is far from being perfect.

Most geophysical inversion schemes use an optimization strategy from the direct methods class (e.g. gradient descent), i.e. they try to use the obtained data to generate a Jacobian matrix that will relate measurements to data points. An alternative to direct inversion methods may be found in evolutionary optimization methods. These methods typically do not attempt to create a direct mapping of input to output but try by a systematic search procedure to identify the objective function minima. Although not trivial to understand, these methods often guarantee convergence to the global minimum (while direct methods may converge to local minima). Genetic algorithms (GA; see for example Holland[6]) are perhaps the most known members of this class of optimization algorithms that also includes simulated annealing.

In geophysics, GA are relatively rarely used. Stoffa and Sen[7] were one of the first to use GA for seismic data inversion. Chunduru et al.[8] applied GA for inversion of resistivity data, followed later by others. Furman et al.[9] used GA for ERT experimental design. One of the main drawbacks of GA is that it requires significantly larger computational resources than direct inversion. This is primarily due to the need to simulate the process of interest (e.g. electrical flow) many times. For geophysical inversion, where the result is often required at very high spatial resolution, this is a significant limitation. Chunduru et al.[8] for example used splines to interpolate between a subset of points which parameter values were changed in the GA. With respect to this problem, the work of Schwarzbach et al.[10] is of special interest. They managed to solve the resistivity inverse problem for a large number of parameters primarily by parallelizing the GA code.

Inspired in part by the idea of Chunduru et al.[8] to correlate adjacent model cells, we suggest here to use an hierarchical approach in GA inversion of resistivity data (the approach can generally be used for any geophysical process). Therefore, the purpose of this paper is to test the concept of hierarchical refinement of the spatial resolution during inversion in order to obtain a more accurate solution while reducing the computational effort typically required for an inversion using GA.

2 COMPUTATIONAL METHODS

2.1 Genetic algorithm and objective function

Genetic algorithms, the most basic class of evolutionary optimization algorithms, were initially developed by John Holland in the 1960-1970's (Holland, 1975). The general idea is to mimic the evolutionary principle of "survival of the fittest" into mathematical notation and to use it in search for an optimal set of parameters describing a system of interest. In brief, a GA includes an elite group that is composed of the best performing sets of parameters, a set of mechanisms that can create new solutions based on the members of the elite group, a mechanism to simulate the process of interest (e.g. electrical current flow) for a given set of parameters (e.g. resistivities), and a mechanism that evaluates and sorts solutions. In evolutionary optimization language, a set of parameters is called a *chromosome*, and each parameter is called a *gene*. In each evolutionary generation, N possible solutions are generated (together with the elite group this is the *population*), each of these N solutions is evaluated, and all solutions are sorted according to their fitness. The best N_e solutions are the new elite group that is used in the next generation to generate new chromosome proposals.

In this work, we have used three mechanisms for generating new possible solutions, namely *gene exchange*, *chromosome perturbation*, and *complete randomness*. The *gene exchange* process uses two randomly selected solutions from the elite group and exchanges up to N_g genes between them to generate two new chromosomes. In total N_x new chromosomes (*mutations*) are created by that manner in each generation. The chromosome perturbation process randomly selects a chromosome from the elite group and performs up to N_p perturbations in this chromosome. In total N_a mutations of this type are generated in each generation. Last, the complete randomness mechanism generates N_r totally new chromosomes (by random population of the genes) in each generation. The purpose of this mechanism is to reduce the chances of the algorithm to get trapped in local minima of the objective function. It is important to note that the optimization of the algorithmic settings is beyond the scope of this paper and will be the topic of future investigations. The GA was implemented in Matlab (The Math Works) without using the GA toolbox of Matlab.

Unlike most GA schemes, we have used a discrete set of possible electrical resistivity (ρ) values (i.e. $\log_{10}\rho$ can be 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, or 4.0). This allowed reducing the computation time (due to reduction of the parameter space), and opened the way for another level of hierarchy in the parameter space (that is not yet implemented). In addition, a unique priority code was provided to each model cell. This code was used to guide the GA to spatial regions of interest. For example, model cells close to the surface received a higher priority in early generations of the GA. This higher priority was translated to probabilities for exchanging or perturbing genes. The subsurface was divided into three priority regions: priority 1 was assigned to depths up to 5 m below the electrodes, priority 2 was assigned to depths from 5 to 10 m below the electrodes, and priority 3 was assigned to the remainder of the domain. Probabilities for alternation were set at ratios of 10:2:1, 5:19:1, and 1:1:2 for priority regions 1 through 3, respectively, for 50, 40, and 10 percent of the total number of generations, respectively.

The fitness of each chromosome is typically quantified by the mismatch between measured and modeled data. In this study, the focus is on the inversion of electrical resistivity data, which is a notoriously ill-posed problem. Therefore, we decided to use a fitness value that consists of two components. The first component is the sum of squared residuals between measured and modeled transfer resistance. These transfer resistances were obtained by solving the electrical forward problem with CRMOD[11]. This model is a 2.5D finite-element-based model solving the Poisson equation that describes electrical flow in the subsurface. The second component is a regularization term that consists of a scaling parameter and an operator that calculates the first derivative of the proposed electrical resistivity distribution. The scaling parameter weights the importance between obtaining an adequate model fit and a reasonably smooth solution and it was set to 0.01 for all model runs based on preliminary investigations.

2.2 Hierarchical approach

As suggested in the introduction, the hierarchical approach may lead to a reduction of the number of generations required by the GA to reach optimality. We considered two levels of hierarchy, in the spatial space and in the parameter space. The later is not implemented in this paper. The basic principle that applies for both cases is that by searching for optimality in coarse resolution (in either spatial or parameter space) and moving to higher resolution with the obtained solution a significant computational effort can be saved. For the parameter space case, the general idea is that at early generations all parameters can vary in wide range (but allowing discrete values in that range), and after optimality is reached the search can be refined to smaller range but in higher resolution.

In the spatial space, we implemented here four spatial resolutions. Optimality is first searched for the coarser resolution, and after it is obtained the optimal solution is downscaled to the next resolution, forming the initial elite group for that level. The details of the four levels considered are listed in Table 1.

Level	Resolution	N	N_e	N_x	N_a	N_p	N_r	Generations
I	3 by 6	50	10	18	18	3	4	200
II	6 by 12	50	10	18	18	7	4	1,000
III	15 by 30	50	10	18	18	45	4	1,000
IV	30 by 60	50	10	18	18	180	4	1,000

Table 1 : GA parameters for the four different spatial levels

3 RESULTS AND DISCUSSION

3.1 Case study

Consider a synthetic case for evapotranspiration from an initially wet hydraulically homogeneous subsurface of 30 m width and 15 m depth. The evapotranspiration creates in the subsurface two distinct regions: a wet region characterized by an electrical resistivity of ρ = 100 Ω·m, and a dry region characterized by an electrical resistivity of ρ = 1000 Ω·m. The electrical resistivity distribution is depicted in Figure 1. Twenty one electrodes are located at 1 m separation at the central part of the domain. To obtain a set of synthetic ‘measurements’, 199 ERT measurements were simulated with CRMOD using a combination of Wenner, Schlumberger and dipole-dipole arrays. In the following, these simulation results are used as measurements in the GA algorithm.

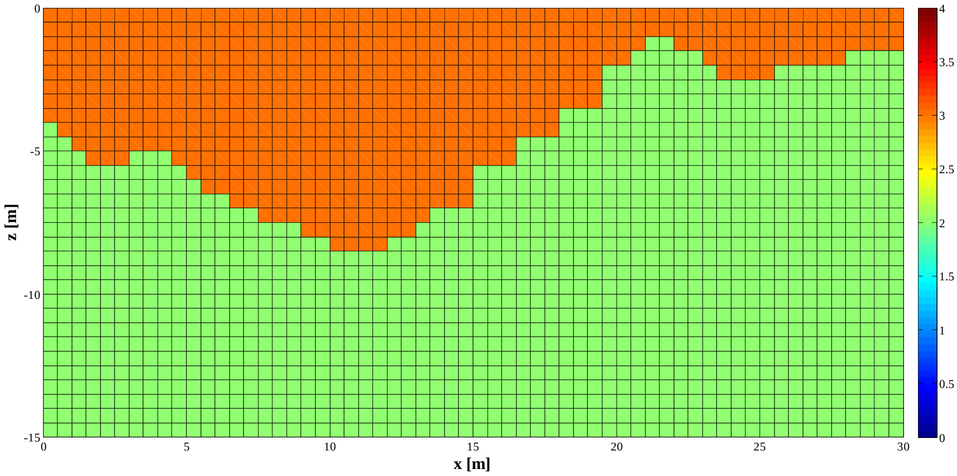

Figure 1: Layout of case-study. Cell size is 0.5 by 0.5 m and electrodes are located at the surface between x = 5 and x = 45 m at 1 m separation. Color bar indicate resistivity in $\log_{10}(\Omega m)$.

3.2 Solutions without using a hierarchical approach

We start by presenting the solutions obtained for each resolution when starting from scratch (Figure 2). In these cases, the initial population is generated randomly. For levels II through IV the obtained result are presented for 1,000 generations, while for the coarsest level I the results are only presented for 200 generations. Figure 2 (lower right) presents the behavior of the objective function for these solutions. Note that although a single run is presented for each case, additional runs (not presented) showed similar behavior.

One can note that some proximity to the true solution was obtained for the coarse resolution (level I). The correct resistivity was obtained for the first layer below the electrodes where sensitivity is highest. Slightly lower resistivity was obtained for the right near surface corner, where the resistive layer is thinner. For the finer resolutions (6 by 12 and 15 by 30), the runs consisting of 1000 generations give at best a clue for the true resistivity distribution (higher resistivities near the surface, somewhat shifted to the left). Also, it is important to note that all cases underestimate the resistivity at deep regions. The number of forward model runs in each run of the GA are 8,000 for level I and 40,000 for each of the other levels (out of 9^{18}, 9^{72}, 9^{450}, and $9^{1,800}$ possible combinations for levels I through IV, respectively). Clearly, only a small fraction of the solution space was explored.

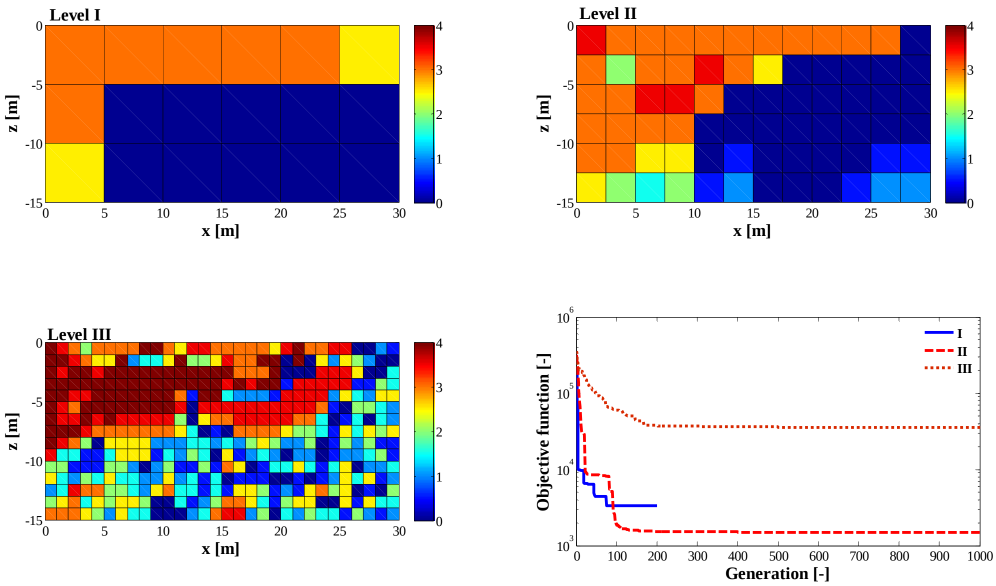

Figure 2: Solutions for three resolutions starting from random guess – level I (top-left), II (top-right), III (bottom-left), and objective-function behavior (bottom-right). Color bars indicate resistivity in $\log_{10}(\Omega m)$.

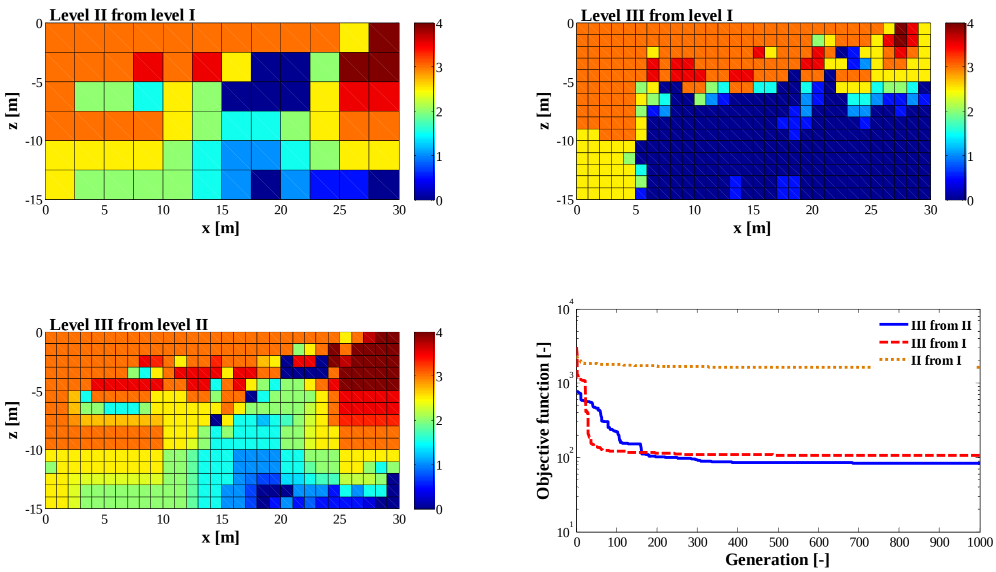

Figure 3: Solutions for three resolutions starting from lower level solution – level II, starting from level I (top-left); level III, starting from level II (top-right); level III, starting from level I (bottom-left); and objective function behavior (bottom-right). Color bars indicate resistivity in $\log_{10}(\Omega m)$.

3.3 Hierarchical solutions

Application of the proposed hierarchical approach leads in most cases to significantly better solution as can be seen both visually and numerically in Figure 3. This is not the case for level II starting for level I, where no significant improvement was obtained. However, looking at the obtained results for level III (both from level I and especially II) indicate a solution that is much closer to the known resistivity distribution shown in Figure 1. This can clearly be further improved with increased number of generations and adjustment of the smoothness constraints.

One can note that here as well, the deeper subsurface is often (although not always) underestimated. The objective function is composed of the model-data mismatch and the smoothness penalty. Although the smoothness penalty is only a small fraction of the model-data mismatch, it seemed during optimization that in many cases it dominated the solution. This is because the variation in the smoothness is of the same order of magnitude as the reduction in model-data mismatch created by alternation of a single model block. In future investigations, the effect of this model regularization will be further explored, perhaps following the multi-objective optimization strategy proposed by Schwarzbach et al.[10].

Of interest is the objective function value obtained for level III. For the direct approach the objective function obtained after 1,000 generations was approximately 35,000 (Figure 2). Using either of the two hierarchical paths (III from I and III from II) the objective function obtained was approximately 100 (Figure 3) – over two orders of magnitude lower. Although this was obtained after 1,200 and 2,000 generations, one can note from Figure 3 that the transition between levels could have been made after 100-200 generations only. That is, the low objective function value of 100 could be obtained after only few 100’s of generations (with a population of 50). Although comparison is not direct, this is a considerable reduction in computation time compared to Schwarzbach et al.[10] who used over 8,000 generations for a population of over 2,000.

4 SUMMARY AND CONCLUSIONS

Using a relatively simple GA, we demonstrated the possibility to use a hierarchical approach to significantly reduce the computational effort required for GA inversion of resistivity data. We presented the behavior of the inversion algorithm for three different resolutions and then to combinations of the three, using the ending point of one solution as the starting point of the other. For the specific case presented, computational effort was reduced by several orders of magnitude. The hierarchical approach was demonstrated so far only in the spatial space, but can also be applied to the parameter space. Although still very slow compared to gradient based algorithms this makes inversion through hierarchical evolutionary algorithms realistic. The advantages of such algorithms, and perhaps primarily the ability to locally remove stabilizing constraints, brings ERT specifically and geophysics in general a step closer to be a more quantitative tool for subsurface characterization and monitoring.

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