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A 3-DIMENSIONAL FINITE ELEMENT MODEL OF THE CIRCULATION IN THE BAY OF BISCAY CONSIDERING TIME-VARYING WIND FIELDS.

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Summary: The goal of this work is to study the wind-induced circulation in the Bay of Biscay using a stabilized 3-dimensional finite element model, HELIKE[1,2]. This model is based upon the incompressible Navier-Stokes equations for geophysical fluids. It uses the Coriolis acceleration, turbulence, bottom friction, wind stresses, density gradient (baroclinic term) and free surface height (barotropic term). The latter is obtained by means of a kinematic equation, without the need of height-averaging. A stabilization method allows for the use of the same shape functions for velocity and pressure.

1 NUMERICAL MODEL

1.1 Basic Equations

Helike is numerical model for solving the Navier-Stokes equations via the finite element method. Even though its primary use is in oceanographic applications, it can be used as a general purpose CFD code with the proper parameters. The model solves the equation system for pressure and velocity components, and uses these in the decoupled free-surface equation. Consider the incompressible Navier-Stokes system:

$$\partial_t u + (u \cdot \nabla)u + k \times u + \nabla p - \partial_x \nu_H \partial_x u - \partial_y \nu_H \partial_y v - \partial_z \nu_V \partial_z w = -g\nabla_H \eta - g\nabla_H \left[\int_z^\eta (\rho - \rho_0)/\rho_0 d\varsigma \right] \quad (1)$$

$$\partial_x u + \partial_y v + \partial_z w = 0 \quad (2)$$

$$\partial_t u + u\partial_x \eta + v\partial_y \eta = w \quad (3)$$

Where u, v and w are the velocity components; p is the pressure; ν_H and ν_V are the horizontal and vertical diffusion coefficients; η is the free surface height and k are the coriolis acceleration components.

1.2 Boundary Conditions

Different boundary conditions are imposed on our domain, depending on the simulation we are running. In this paper we will only mention those concerning our study. If the 3-Dimensional domain is decomposed in three parts we have:

$$\Gamma_s = S \times 0 \qquad \text{(Surface)}$$

$$\Gamma_b = \{(x,y,z) \in \mathbb{R}^3/(x,y) \in S, -\mathcal{H}(x,y) = z\} \qquad \text{(Bottom)}$$

$$\Gamma_b = \{(x,y,z) \in \mathbb{R}^3/(x,y) \in \partial S, -\mathcal{H}(x,y) \leq z \leq 0\} \qquad \text{(Lateral boundary)}$$

The boundary conditions applied to these parts of the domain are:

Impermeable bottom:

$$w + u\frac{\partial \mathcal{H}}{\partial x} + v\frac{\partial \mathcal{H}}{\partial y} = 0 \text{ on } \quad \Gamma_b \tag{4}$$

Tangent Wind Stresses:

$$\nu_V\left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}\right) = \left(\tau_s^x, \tau_s^y\right) \coloneqq \frac{\rho_a}{\rho_0} C_s (U_{10}^2, V_{10}^2)^{1/2} (\mathrm{U}_{10}, \mathrm{V}_{10}) \text{ on } \Gamma_s \tag{5}$$

where ρ_a is the air density, C_s is the wind drag coefficient and *(U_{10}, V_{10})* are the wind velocity components 10m. above the sea surface. Also a Zero normal stress condition is imposed on the surface:

$$\nu_V \frac{\partial w}{\partial z} = 0 \; on \; \Gamma_s \tag{6}$$

Inflow on walls:

$$u = u_i \; on \; \Gamma_i \tag{7}$$

where u_i is the inflow velocity.

Zero normal velocity on lateral walls:

$$n_x u + n_y v = 0 \; on \; \Gamma_w \tag{8}$$

Finally, initial velocity and free surface conditions are imposed in the linear case:

$$u(x,y,z,0) = u_0\,(x,y,z), \forall (x,y,x) \in \Omega \tag{9}$$

and

$$\eta(x,y,0) = \eta^0(x,y), \qquad \forall (x,y) \in \Gamma \tag{10}$$

1.3 Numerical Approximation

After applying boundary conditions, we multiply the equations by test functions and integrate over the whole domain to obtain the following:

Velocity-Pressure system:

$$\begin{aligned} &\int_\Omega \partial_t u\tilde{u} d\Omega + \int_\Omega (u\cdot\nabla)u\tilde{u}d\Omega + \int_\Omega (k\times u)\tilde{u}d\Omega + \int_\Omega \nabla p\tilde{u}d\Omega \\ &- \int_\Omega \left(\partial_x \nu_H \partial_x \tilde{u} - \partial_y \nu_H \partial_y \tilde{u} - \partial_z \nu_V \partial_z \tilde{u}\right) d\Omega \\ &= \quad -g\int_\Omega \nabla_H \eta\tilde{u}d\Omega - g\int_\Omega \nabla_H \left[\int_z^\eta (\rho-\rho_0)/\rho_0 d\varsigma\right] \tilde{u}d\Omega + \int_{\Gamma_s} \left(\tau_s^x \tilde{u} + \tau_s^y \tilde{v}\right) d\Gamma_s \\ &+ \int_{\Gamma_s} \left(\tau_b^x \tilde{u} + \tau_b^y \tilde{v}\right) d\Gamma_s + \int_{\Gamma_s} n_\nu \cdot \nabla w\tilde{w} d\,\Gamma_s \end{aligned} \tag{11}$$

Continuity equation:

$$\int_\Omega (\nabla\cdot v)\tilde{q}d\Omega = 0 \tag{12}$$

Free Surface Equation:

$$\int_{\Gamma_s} \partial_t \eta\tilde{\mu} d\Gamma_s + \int_{\Gamma_s} u\,\partial_x \eta\tilde{\mu}\Gamma_s + \int_{\Gamma_s} v\,\partial_y \eta\tilde{\mu}\Gamma_s = \int_{\Gamma_s} w\tilde{\mu}\Gamma_s \tag{13}$$

1.4 Stabilization Method

The goal of the stabilization method is to stabilize the pressure [3]. The idea is to introduce, as a new unknown of the problem, the projection of the pressure gradient onto the velocity space and to add to the incompressibility equation the difference between the Laplacian of the pressure and the divergence of this new vector field.

The new unknown is the projection $r_h^{n+1} = (r_1^{n+1}, r_2^{n+1}, r_3^{n+1})$ in $L^2(\Omega)$ of the pressure gradient ∇q_h^n weighted in each element by the parameter$\sqrt{\alpha_K}$. This new equation is written as:

$$\int_\Omega r_h^{n+1} \tilde{s}_h \, d\Omega - \sum_{K\in\Omega_K} \int_K \sqrt{\alpha_K} \nabla q_h^{n+1} \tilde{s}_h d\Omega = 0 \tag{14}$$

where r_h^n is the projection of the pressure gradient; α_K is an element wise stabilization parameter; $\int_\Omega$ denotes the integration over each element of the domain; and K refers to the elements of the mesh. The weak form of the equation is:

$$\int_\Omega (\nabla \cdot u_h^{n+1}) \tilde{q}_h d\Omega + \sum_{K\in\Omega_K} \int_\Omega \alpha_K \nabla q_h^{n+1} \nabla \tilde{q}_h d\Omega - \sum_{K\in\Omega_K} \int_\Omega \sqrt{\alpha_K} r_h^{n+1} \nabla \tilde{q}_h d\Omega \tag{15}$$

The coefficient α_K is obtained with this expression:

$$\alpha_K = \left(c_1 \frac{\nu}{h_k^2} + c_2 \frac{V_k}{h_k} + \frac{1}{\Delta t} \right)^{-1}, \forall K \in \Omega \tag{16}$$

where h_k is the element size; V_k is a characteristic element velocity and the constants are $c_1 = 12$ and $c_2 = 6$ [4].

1.5 Turbulence closure models

1.5.1 Horizontal turbulent viscosity

Two models are used for the horizontal turbulence, constant values and the Smagorinsky parameterization. In the latter, the horizontal coefficient ν_h is defined as:

$$\nu_h = C_{mo} h_x h_y D_T \tag{17}$$

where $\boldsymbol{C_{mo}}$ is an adimensional viscosity coefficient, $\boldsymbol{h_x}$ and $\boldsymbol{h_y}$ are the size of the horizontal discretization in the *X* and *Y* direction and:

$$D_T = \left((\partial_x u)^2 + (\partial_x v)^2 + \frac{1}{2} \left(\partial_y u + \partial_x v \right)^2 \right)^{1/2} \tag{18}$$

is the magnitude of the velocity of the strain tensor.

1.5.2 Vertical turbulent viscosity

Besides the constant value for the vertical turbulence coefficient, three models are used which are based on the Richardson number:

$$R_i = N^2/M^2 \tag{18}$$

where $N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}$ is the frequency of the vertical oscillations and $M^2 = \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2$. is

The majority of the models used depend on an algebraic expression that is written as:

$$\nu_V = \nu_0 + \nu_1/(1+\beta R_i)^n \tag{19}$$

Depending on the values chosen for ν_1, ν_0, n and β, these models are known as Pacanowsky-Philander[5], Gent[6], and Munk & Anderson[7].

2 IMPLEMENTATION AND APPLICATION TO CIRCULATION IN THE BAY OF BISCAY

2.1 HELIKE Numerical Model

The Helike model consists of two parts. The first part is the pre and post processing software GID, which allows us to build the model geometry, apply boundary conditions and run the calculation with a very user friendly GUI. After the calculations have been performed, the results can be viewed in the GID's post processing mode. The second part is the core of the model, which implements the formulation described in section 1 using FORTRAN programming language and writes the results to files that can be read by GID.

This system has many advantages. The most important is the finite element formulation, as it permits us to deal with problems of a complex geometry. Also, we note the use of a kinematic free surface equation. Besides, the coupling of the FORTRAN code with GID saves the user from learning complex programming concepts, and only focus on the physical and numerical aspects of the problem.

2.2 Domain of Study

The circulation in the Bay of Biscay it caused mainly by the winds and the variability of the density [8,9]. In this study we will only consider the influence of the wind.

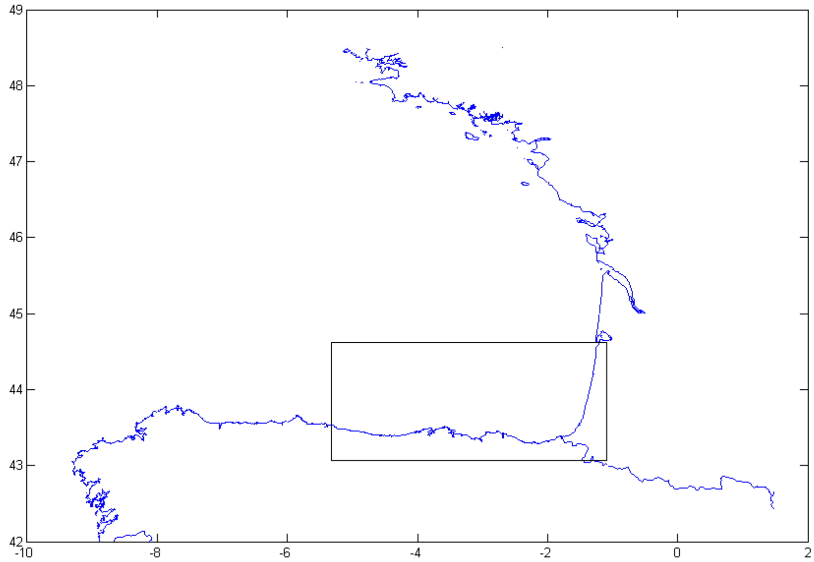

Figure 1. Northern coast of Spain. The rectangle shows our domain of study.

The region being studied is located between the longitudes 1 and 4 W, and the latitudes 43N and 44N. The mesh consists of 16875 nodes and 11208 prismatic elements. The meshing process begins with a surface mesh, which is then projected to the bottom to create the volumetric mesh.

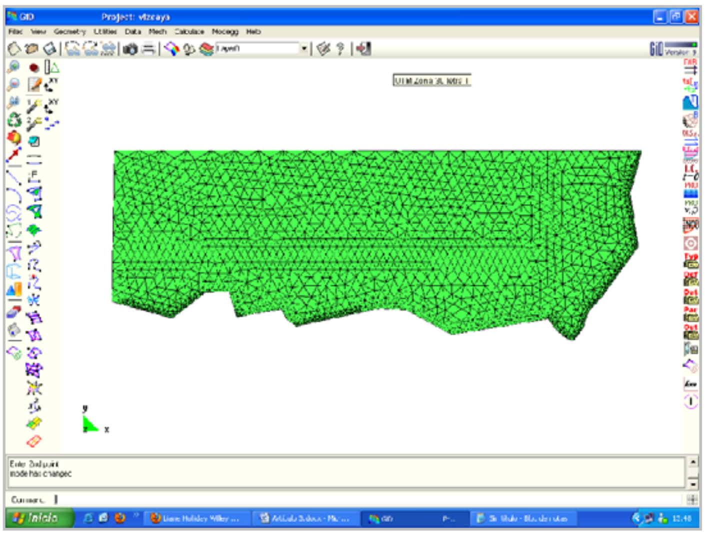

Figure 2. GID user interface. Mesh of the domain of study

2.3 Forcing data and boundary conditions.

The wind velocity vectors have been interpolated for each node of the mesh in an hourly basis, the total simulation time is 48 hour. The period of time from 15-16/11/2008 has been chosen because it shows enough variability to test and calibrate our model. We also use a stationary velocity field obtained by applying a constant wind field in the surface. We have used an implicit time integration scheme, solving the systems of equations via the conjugate gradient method.

2.4 Results and conclusions.

We have presented a numerical model to solve the Navier-Stokes equations and applied this to the study of the wind induced circulation in the Bay of Biscay. This model incorporates wind data from Meteogalicia operational models and uses it to calculate the wind friction on the surface.

Our future research will be directed towards the development of an operational model. We'll incorporate values from actual measures (atmospheric pressure, density, free surface) and other models such as ROMS[10]. Also a major concern is to speed up the calculations by means of implementing new time integration schemes and the use of parallel solvers.

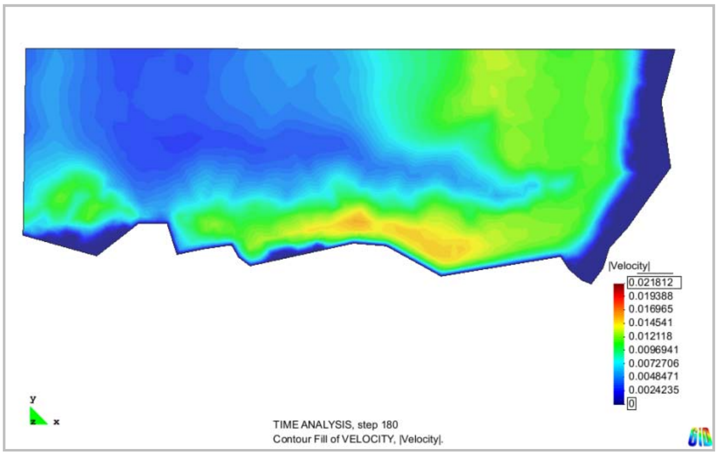

Figure 3. Contour fill of the velocities on the surface at 15/11/2008, 3:00 pm.

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