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NUMERICAL METHODS FOR WEAKLY NONLINEAR STABILITY ANALYSIS OF SHALLOW WATER FLOWS

Mohamed S. Ghidaoui[*], and Andrei A. Kolyshkin[†]

[*] Hong Kong University of Science and Technology
Clear Water Bay, Kowlooon, Hong Kong, China
e-mail: ghidaoui@ust.hk, web page: http://ihome.ust.hk/~ghidaoui/

[†] Riga Technical University
1 Meza street LV1007 Riga, Latvia
e-mail: akoliskins@rbs.lv

Summary. Large scale two-dimensional structures in shallow water flows are believed to appear as a result of hydrodynamic instability of the flow. Complex eddies in the wake of islands can lead to poor water quality by trapping sediments and pollutants. Linear stability theory is widely used in such cases to predict the onset of instability. The evolution of the most unstable mode above the threshold is often decsribed in fluid mechanics by the methods of weakly nonlinear theory in an attempt to explain the development of instability in real wake flows. Direct application of the method of multiple scales to problems in fluid mechanics usually leads to amplitude evolution equations such as complex Ginzburg-Landau equation (CGLE). The coefficients of the CGLE are expressed in terms of integrals containing the solutions of several linearized problems.
The shallow water equations under the rigid-lid assumption can be written in the form [1] $L\psi = 0$, where ψ is the stream function of the flow. Expanding the stream function in powers of ε of the form

$$\psi(x,y,t) = \psi_0(y) + \varepsilon\psi_1(x,y,t) + \varepsilon^2\psi_2(x,y,t) + \varepsilon^3\psi_3(x,y,t) + \dots, \quad (1)$$

introducing the „slow" time $\tau = \varepsilon^2 t$ and longitudinal variable $\xi = \varepsilon(x - c_g t)$, where c_g is the group velocity, and using the method of multiple scales one can obtain the following equations: $L_0\psi_1 = 0,\ L_0\psi_2 = f(\psi_0,\psi_1),\ L_0\psi_3 = g(\psi_0,\psi_1,\psi_2)$. The CGLE is derived in [1] by applying the solvability condition to the function g. It is shown in [1] that in order to obtain the solutions ψ_1 and ψ_2 one needs to solve five boundary value problems (one of which is resonantly forced).
In the present paper we describe an accurate numerical method for the solution of the above mentioned five boundary value problems. Collocation scheme based on Chebyshev polynomials is developed in the paper. Two examples are considered: (a) one-component shallow water flow and (b) two-component shallow water flow for the case of large Stokes number. The coefficients of the CGLE are calculated for both cases.

References

[1] A.A. Kolyshkin, M.S. Ghidaoui (2003), Stability analysis of shallow wake flows, Journal of Fluid Mechanics, v. 494, pp. 355 – 377.