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DISCONTINUOUS GALERKIN APPROXIMATION OF TWO-PHASE FLOWS IN HETEROGENEOUS POROUS MEDIA WITH DISCONTINUOUS CAPILLARY PRESSURES

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Summary. The equations governing two-phase immiscible incompressible flows in porous media generally consist of an elliptic Darcy-type equation for the pressure and a nonlinear degenerate parabolic equation with a nonlinear advective term for the saturation. In heterogeneous porous media with discontinuous capillary pressures, the interface conditions are nonlinear and can lead to a nonzero jump at the interface for the pressure and the saturation [1]. In this work, we design and investigate a sequential discontinuous Galerkin (dG) method to approximate the above equations. The interface conditions are enforced weakly through an adequate design of the penalties on interelement jumps of the pressure and the saturation. An accurate reconstruction of the total velocity is considered in the Raviart--Thomas(-Nédélec) finite element spaces [2], together with diffusivity-dependent weighted averages to cope with degeneracies in the saturation equation and with media heterogeneities [3]. The proposed method is assessed on one-dimensional test cases exhibiting rough solutions, degeneracies, and capillary barriers. Stable and accurate solutions are obtained without limiters.More details can be found in [4].

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