Flow in natural porous media is often in the laminar Stokesian regime. However, in many applications, both in subsurface materials and in industrial applications of porous media, flows contain significant inertial components. The observation that the inertial component of flow can dramatically influence the pressure-fluid flux response in porous media (i.e., the flows become non-Darcian) has been known for decades; however, only a very few microscale analyses have been conducted that help explain this observation. Here, we report on microscale simulations and associated upscaling for the problem of flow in porous media to the case of non-Darcy flow regimes. In particular, it is found the structure of the flow field dramatically changes in the transition from non-inertial to inertial flows, with the formation of jets and vorticies being evident at high Reynolds numbers. These structures appear to be responsible for the changes in the pressure-fluid flux relation during non-Darcy flow.