Numerical Approximation of Two-Phase Compressible Flow in Porous Media by the Weighted Interior Penalty Discontinuous Galerkin Method

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We present a new numerical scheme for simulating immiscible compressible air-incompressible water isothermal flow through heterogeneous porous media.

It is based on the recently introduced Amaziane-Jurak fractional flow formulation which generalizes the well known Chavent-Jaffre global pressure-saturation formulation for incompressible flow. It assumes that air mass density satisfies the ideal gas law and that it is proportional to air pressure. That new formulation is exactly equivalent to the phase pressure-water saturation formulation and is an appropriate choice for the case of small operating pressures as they appear in air-water system in groundwater hydrology [1]. The governing equation for pressure is a nonlinear parabolic equation and the equation for saturation is a nonlinear convection-diffussion equation.

The key ingredient of our new numerical scheme is Weighted Interior Penalty discontinuous Galerkin (WIP dG) method applied to the diffusive terms of both equations. WIP dG method is robust to non-smooth and possibly vanishing diffusivity [2]. It penalizes jumps of discrete solution by factor proportional to the harmonic mean of neighbouring normal diffusivities instead by arithmetic mean as standard IP dG methods do.

Computational results show that the proposed numerical scheme is a natural fit for porous media with different rock types where saturation and pressure may be discontinuous.

References

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[2] Ern A., Stephansen A., Zunino P.: *A Discontinuous Galerkin method with weighted averages for advection-diffusion equations with locally small and anisotropic diffusivity*, IMA J. Numer. Anal., published online on March 20, 2008. doi:10.1093/imanum/drm050