Comparison of cell-centered and node-centered formulations of a high-resolution well-balanced finite volume scheme: application to shallow water flows

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Finite volume (FV) methods for solving hyperbolic conservation laws on unstructured meshes are known for a long time. There are mainly two basic formulations of the FV method: cell-centered and node-centered. A detailed comparison is presented between the two formulations using a FV high-resolution algorithm on triangular unstructured grids, which was developed for the simulation of unsteady, two-dimensional shallow-water flows over arbitrary topography with wetting and drying.
In both formulations we properly utilize Roe's approximate Riemann solver to compute the numerical fluxes, while second-order spatial accuracy is achieved with a MUSCL reconstruction technique, using slope limiting to control the total variation of the reconstructed field, and an explicit, second order in time, four-stage Runge–Kutta time stepping. Consistent well-balanced second-order discretizations for the topography source term treatment and the wet/dry front treatment will be presented, for both formulations, ensuring absolute mass conservation, along with stable friction term treatment. Both algorithms have the ability of local grid refinement at the beginning of the computational procedure at regions with steep variations of the topography.
The motivation for this comparison between the two approaches was the investigation of whether and in what degree some theoretical advantages of the node-centered approach against the cell-centered one are materialized in enhanced accuracy, robustness and time efficiency. Comparing a node-centered FV formulation and a cell-centered one, on the same mesh, the former has fewer degrees of freedom, about half the total memory foot-print, and more fluxes per unknown. Moreover, the cell-centered discretization has a relative sparse stencil, with each triangle having only three neighbors. The node-centered approach produces smother computational cells, even for highly distorted meshes. Additionally, the larger stencil has the potential for more robust reconstruction techniques and limiting procedures, while, as opposed to cell-centered schemes, treatment of the boundary conditions is assisted by the fact that control volume centers are located precisely on the boundary.
Several benchmark test cases, experimental and field data were used, for both the cell-centered and the node-centered approach, in order to validate and compare the two with respect to their effectiveness and robustness.