Simulation of Soliton-like Waves Generated by Topography with Conservative Fully Discrete Shallow-Water Schemes

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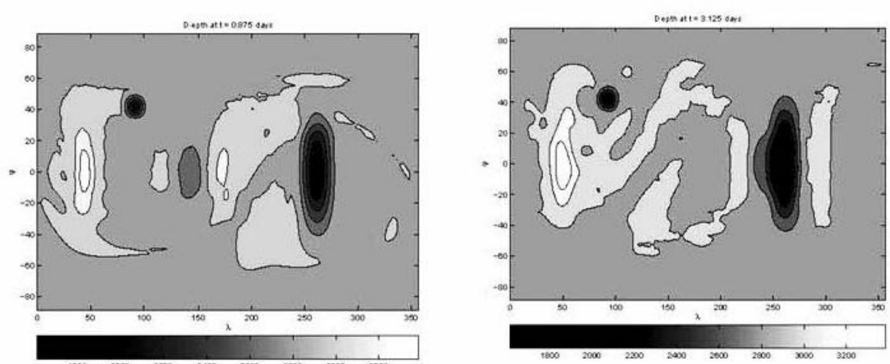
A new numerical method for the simulation of shallow-water model (SWM) flows is suggested [1,2]. The method is based on splitting of the model operator by physical processes and by coordinates. An essential advantage of the method is that it provides fully discrete (both in time and in space) shallow-water schemes of arbitrary approximation orders in space that exactly conserve the mass and the total energy and whose numerical implementation is computationally inexpensive. Moreover, the method permits to resolve the SWM on a whole sphere with the same numerical algorithms (of arbitrary approximation order in space) which are used in a doubly periodic domain on the plane. The quality of numerical schemes are illustrated by comparing the numerical results obtained on the simulation of nonlinear soliton-like waves generated by a model topography. The numerical experiments included testing several finite difference schemes of different approximation order in space. Numerical results obtained on grids of different resolution are also compared. The primary attention was given to the study of time-space structure of the numerical solutions.

We stress that each finite difference scheme exactly conserves the mass and the total energy, but not the potential enstrophy. Since the potential enstrophy is one of the basic invariants of the shallow-water motion, temporary behavior of the potential enstrophy was considered as an important integral characteristic of the schemes' quality in all the experiments. It is shown that the variations

$$\delta J(t) = \frac{\max J(t) - \min J(t)}{\min J(t)} \cdot 100\%$$

of potential enstrophy $J(t)$ in time are within narrow bands and very small - the quantity does not exceed 0.05% on fine ($3° \times 3°$ or better) grids.

A quasi-periodicity and structural stability of the wave observed in Fig.1 for time moments $t = 0.875\, days$ and $t = 3.125\, days$ are due to the schemes conserving the mass and total energy.



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References

[1] Skiba Yu.N. and D.M. Filatov (2009). Simulation of Soliton-like Waves Generated by Topography with Conservative Fully Discrete Shallow-Water Arbitrary-Order Schemes. *International Journal of Numerical Methods for Heat and Fluid Flow*, 19 (8), 982-1007.

[2] Skiba Yu.N. and D.M. Filatov (2008). Conservative Arbitrary Order Finite Difference Schemes for Shallow-Water Flows. *Journal of Computational and Applied Mathematics*, 218 (2), 579-591.