XVIII International Conference on Water Resources
CMWR 2010
J. Carrera (Ed)
CIMNE, Barcelona 2010

CENTRAL WENO FOR SHALLOW WATER EQUATIONS IN CONTRAVARIANT FORMULATION

F. Gallerano†*, G. Cannata† and M. Tamburrino†

† Dipartimento di Idraulica Trasporti e Strade, Sapienza Università di Roma
Via Eudossiana 18, 00184, Roma, Italy

* e-mail: francesco.gallerano@uniroma1.it

Summary. Many authors solve shallow water equations by using high-resolution methods for hyperbolic systems of conservation laws. In this context, Essentially Non-Oscillatory (ENO) and Weighted Essentially Non-Oscillatory (WENO) schemes are the most efficient tools. Flow simulations over computational domains characterized by a complex boundary can be performed by numerical integrations of motion equations on a generalized curvilinear boundary conforming grid. In this approach the domain is greatly simplified, since it is transformed into a fixed rectangular region and the WENO reconstructions are performed directly in transformed space where the computational space step is constant. In order to make use of a general boundary-conforming curvilinear coordinate system, two different strategies can be followed: in the first strategy, motion equations are projected onto directions identified by the Cartesian reference system and are modified by transforming partial derivatives with respect to Cartesian coordinates to partial derivatives with respect to curvilinear coordinates; in the second strategy, motion equations are expressed directly in covariant or contravariant formulation. The original contribution of this work is the definition of a new Central Weighted Essentially Non-Oscillatory scheme for the solution of the shallow water equations expressed directly in contravariant formulation. The proposed central WENO scheme is the extension of the methodology presented by many authors (*e.g.* D. Levy, G. Puppo, G. Russo, "A fourth-order central WENO scheme for multidimensional hyperbolic systems of conservation laws", SIAM J. Sci. Comput., 24(2), 480-506 (2002)) into the context defined by contravariant form of the equations. One of the most important elements of the C-WENO scheme based on this approach involves the advancing from time level tn to time level tn+1 of the cell averaged values of flow variables. The extension of the above mentioned methodology into the contravariant environment implies that the contravariant shallow water equations must be expressed in integral form. An element of novelty presented in this paper regards the definition of a formal integral expression of the shallow water equations in contravariant formulation, in which Christoffel symbols are avoided. The WENO reconstructions are performed by a two dimensional interpolating procedure taking into account the curved coordinate lines; in the computational domain the spatial discretization step is constant: consequently the problems related to negative linear weights on unstructured meshes are overcome. The two dimensional reconstructions have a fifth-order spatial accuracy. A Natural Continuous Extension into a Runge-Kutta solver is involved in a fourth-order time discretization of motion equations. The proposed scheme ensures the satisfaction of the exact C-property.