

## COMPUTATIONALLY EFFICIENT INVERSION OF STEADY-STATE STOCHASTIC MOMENT EQUATIONS OF GROUNDWATER FLOW

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**Summary.** We present the equations satisfied by the sensitivity matrix of the (ensemble) mean hydraulic head under steady-state groundwater flow conditions. These equations have been embedded in a geostatistical inverse procedure to condition zero- and second-order (in terms of the standard deviation of the natural logarithm of hydraulic conductivity,  $Y$ ) approximations of stochastic moment equations of flow on information on hydraulic conductivity and hydraulic heads. Relying on direct calculation of the derivatives of (mean) hydraulic heads with respect to model parameters allows considerable improvement of the methodology originally proposed by *Hernandez et al.* [2003, 2006], not only in terms of accuracy of the inverse solution, but also in terms of the CPU time required. The log-conductivity field is parametrized geostatistically from (uncertain)  $Y$  measurements and unknown values at discrete “pilot point” locations. Whereas prior values of  $Y$  at pilot points are obtained by a variant of kriging, posterior estimates at pilot points are obtained through a maximum likelihood fit of computed to measured heads. The maximum likelihood function includes a regularization/plausibility term penalizing large departures of  $Y$  estimates from their prior information. The relative weight associated with this term and the statistical parameters of the  $Y$  variogram are evaluated separately using model selection criteria to avoid bias and instability. The variance and covariance of head estimates are then obtained *a posteriori* by solving corresponding moment equations. We illustrate our algorithm and procedure by means of a synthetic example. We explore the influence of the order of approximation of the governing mean flow equation on our ability to properly reconstruct the log-hydraulic conductivity and head fields and to identify the statistical parameters of the underlying  $Y$  variogram, the relative weight of the regularization term and the uncertainty associated with the available measurements.