

## **A DESCRIPTION OF LINEAR AND NONLINEAR SOLVER FAILURES AND CURES FOR UNSATURATED FLOW CALCULATIONS**

**Fred T. Tracy**\*

\* Engineer Research and Development Center  
Department of Defense Supercomputing Resource Center  
3909 Halls Ferry Road  
Vicksburg, MS, USA 39180  
e-mail: fred.t.tracy@usace.army.mil

**Summary.** In looking for robust linear and nonlinear solvers for difficult unsaturated flow problems, it was observed that sometimes a BiCG-Stabilized solver for Newton's method sometimes failed. This was in the context of a three-dimensional (3-D) Galerkin finite element method discretization where only tetrahedral elements were used. It was also observed that not only the 3-D solution failed, but also a 2-D finite difference version of the test problem using a direct banded linear solver also failed. The original problem was essentially a 3-D version of the Green and Ampt problem where a dry soil sample had a head applied at the top of the sample, thus generating a moving downward front of water. When the 1-D Green and Ampt problem was then tested using the finite difference method, the direct tridiagonal system of equations did not fail; but unless remedies were applied, the nonlinear solution process still failed. This paper will describe the cause and different cures of the problem. When Newton's method is used to linearize the nonlinear problem, elements close to the sharp-moving front have both a positive and negative contribution to the main diagonals of their respective stiffness matrices. It was discovered that sometimes the negative contribution dominated, and some of the overall main diagonal terms of the assembled stiffness matrix were negative. The instant these negative values occurred, both the iterative and banded direct solvers failed. This paper will show from equations and example calculations three solutions: (1) use the traditional method of reducing the time-step so the time term provides more positive support, (2) have the relative hydraulic conductivity vary linearly inside each element as compared with being considered constant, and (3) do some Picard iterations before the full Newton method is done, as this avoids the negative contributions from Newton's method.